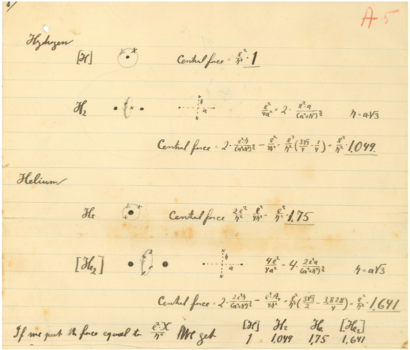
Universality results in quantum gravity

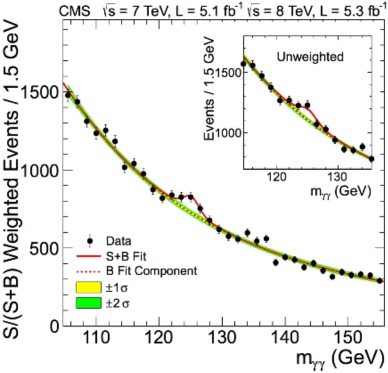
Pierre Vanhove

University of Stony Brook

based on work done with

N.E.J. Bjerrum-Bohr, John Donoghue



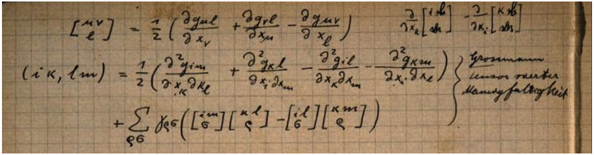


This year we celebrated the 100 years anniversary of Niels Bohr's first paper on quantum mechanics

Quantum mechanics and quantum field theory are tremendously successful culminating with the discovery of the Higgs boson

General relativity is amazingly successfull

In 2 years time (Nov, 25 2015) we will celebrate the the 100 years anniversary of Einstein's general relativity paper.



(Zürich notebook - circa 1912/1913)

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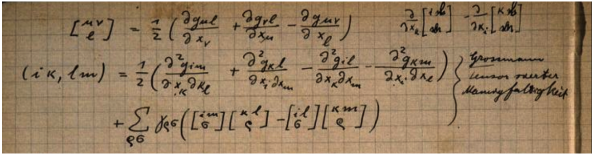
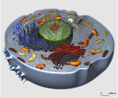


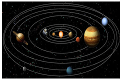
(Zürich notebook - circa 1912/1913)

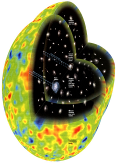
General relativity is remarkably successful theory in the weak field limit (our solar system) and stronger fields regime (binary pulsars).

GR is the standard paradigm for spacecraft navigation and astrometry, astronomy, astrophysics, cosmology and fundamental physics

In fact we have rather poor understanding of the violation of the validity of GR



$10^{-35}\,m$ $10^{-6}m$ $1m$ $10^{9}m$ $10^{19}m$ $10^{21}m$ $10^{27}\,m$

??? poorly tested very good knowledge pretty good knowledge no precise datas poorly tested

Adapted from the ESA Fundamental Physics Roadmap (2010)

lab experiments

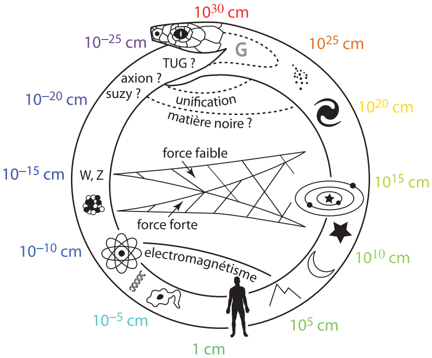

spatial probes

astronomy astrophysics

cosmology

The importance of gravity

Gravity couples to all matter and energy type



Gravity couples to any scales from very short to very large scales

The importance of gravity

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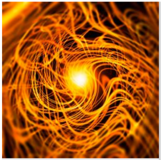
News

Gravity shows its helpful side

Theoretical study shows that the force can ease quantum calculations.

Geoff Brumfiel



Gravity is usually an obstacle to a theory of everything

MEHAU KULYK/SCIENCE PHOTO LIBRARY

Gravity is unruly. It can throw theorists' equations into chaos, and has proved a stumbling block to the creation of a single 'theory of everything'. But an analysis now shows that gravity may at least make some fundamental calculations more manageable.

David Toms, a theoretical physicist at Newcastle University, UK, has found that gravity seems to calm the electromagnetic force at high energies. The finding could make some calculations easier, and is a rare case in which gravity seems to work in harmony with quantum mechanics, the theory of small particles. His paper is published today in *Nature*[1].

But don't get too excited: that elusive theory of everything is not just around the corner. Not everyone thinks that the calculations will stand up to scrutiny. Given physicists' "dicey" understanding of the

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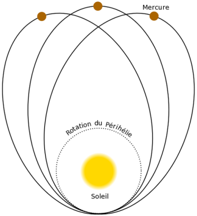
The general relativity effects

Non-linearities of Einstein's general relativity corrects Newton's potential

$$V(r) = -\frac{G_N m_1 m_2}{r} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + \cdots$$

This is important for Mercury perihelion precession



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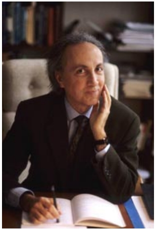
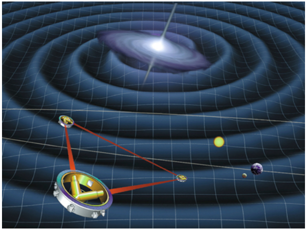
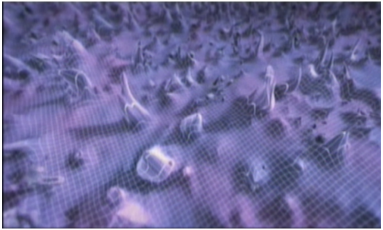
Understanding the systematics of the post-Newtonian corrections is extremely important for detection of gravity waves by the next generations of interferometers





Quantum gravity effects?



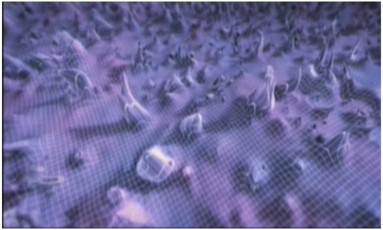
What would be quantum gravity corrections to Newton's potential?

Quantum ambiguity of the order of the Compton wave-length

$$\lambda = \frac{\hbar}{m_1 + m_2}$$

$$\frac{1}{(r \pm \lambda)^2} \sim \frac{1}{r^2} \mp \frac{2\hbar}{(m_1 + m_2) r^3} + \cdots$$

Quantum gravity effects?



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$$V(r) = -\frac{G_N m_1 m_2}{r} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + Q\,\frac{G_N^2 m_1 m_2 \overbrace{(m_1 + m_2)\lambda}^{\hbar}}{r^3} + \cdots$$

Classical and Quantum gravity effects together

A quantum effect is naturally associated to classical GR contributions

$$V(r) = -\frac{G_N m_1 m_2}{r} + C\,\frac{G_N^2 m_1 m_2 (m_1+m_2)}{r^2} + Q\,\frac{G_N^2 m_1 m_2 \hbar}{r^3} + \cdots$$

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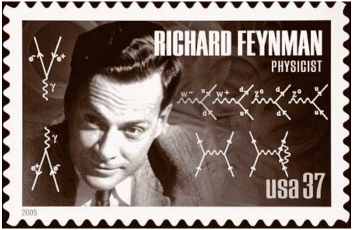
In this talk I will explain that modern amplitudes technics and the relation $gravity = (yang - mills)^2$ and will make this computation *very simple* with *important physics insights*

Some good Advices

Don’t Modify Gravity–Understand It!
Nima Arkani-Hamed

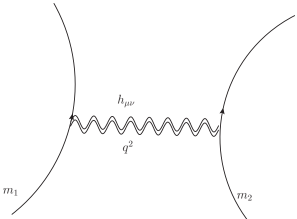
I just put 1.795372 and 2.204628 together.
And what does that mean?
Four!
Doctor Who

Perturbative technics



Classical Newton's potential from a tree-level amplitude

$$V(q) = -\frac{G_N m_1 m_2}{q^2} = \frac{1}{4m_1 m_2}\mathfrak{M}^{non-rel}_{tree}$$



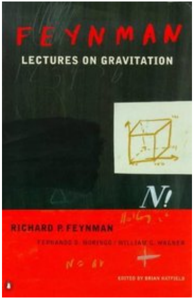
Quantum gravity amplitudes

- Starting from the Einstein-Hilbert action

$$\mathcal{S} = \frac{2}{32\pi G_N} \int d^4x \, \sqrt{-g} \, (\mathcal{R} + g^{\mu\nu} T_{\mu\nu})$$

- Perturbation around the flat space-time $\kappa_{(4)}^2 = 32\pi G_N$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_{(4)} \, h_{\mu\nu}$$



- One can try to treat quantum gravity as an ordinary quantum field theory
- Propagating massless spin 2 particle : the graviton
- Similar to gauge theories with *huge gauge symmetry* from diffeomorphism invariance

Corrections to Newton's potential from amplitudes

We want to derive the corrections to the non-relativistic Newton's potential

$$V(r) = -\frac{G_N m_1 m_2}{r} + C\,\frac{G_N^2 m_1 m_2 (m_1+m_2)}{r^2} + Q\frac{G_N^2 m_1 m_2 \hbar}{r^3}$$

Take a Fourier transform

$$V(q) = \int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}} V(r)$$

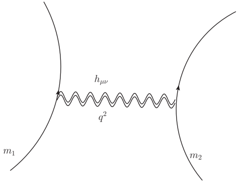
to get the potential

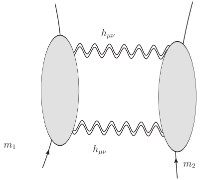
$$V(q) = \frac{G_N m_1 m_2}{q^2} + C\frac{G_N^2 m_1 m_2 (m_1+m_2)}{\sqrt{-q^2}} + Q G_N^2 m_1 m_2 \hbar \log(-q^2)$$

Perturbative technics

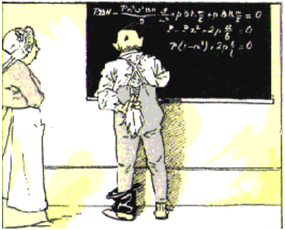
John Donoghue has shown how to get these corrections from loop amplitudes

$$V(q) = \frac{G_N m_1 m_2}{q^2} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{\sqrt{-q^2}} + Q\, G_N^2 m_1 m_2 \hbar \log(-q^2)$$





Classical physics from loops



We want to extract the classical and quantum correction from a one-loop computation

- Quantum corrections of order $\hbar^n$ requires an n-loop amplitude computation

- A loop amplitude can give rise to a classical, ie of order $\hbar^0 = 1$, contribution

Classical physics from loops

What is reason for the appearance of classical contribution at loop order?

At each vertex we have a power of $\hbar^{-1}$ from

$$e^{\frac{i}{\hbar}\int d^4x \mathcal{L}_{int}(x)}$$

For each propagator we get a power of $\hbar$ from

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int d^4k \, \frac{i\hbar}{k^2 - \frac{m^2}{\hbar^2} + i\varepsilon}\, e^{ik\cdot(x-y)}$$

Therefore a graph with V vertices, I propagators and L loops has

$$\hbar^{-V+I} = \hbar^{L-1}$$

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Therefore a graph with V vertices, I propagators and L loops has

$$\hbar^{-V+I} = \hbar^{L-1}$$

But in a non-relativistic limit *mass depend terms* can arise with no $\hbar$

$$\hbar\times\frac{m}{\hbar\sqrt{-q^2}} = \frac{m}{\sqrt{-q^2}}$$

Why is this computation meaningful?

Gravity is plagued by terrible ultraviolet divergences

Can we extract meaningful physical quantities from a *quantum gravity* computation?

At one-loop there is a R^2 counter-term found by ['t Hooft and Veltman]

$$S = \int d^4x |-g|^{\frac{1}{2}} \left[\frac{2}{32\pi G_N} \mathcal{R} + c_1 \mathcal{R}^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

Why is this computation meaningful?

John Donoghue showed that some physical properties of quantum gravity are *universal* being independent of the UV completion

$$V(q) = \frac{G_N m_1 m_2}{q^2} + C\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{\sqrt{-q^2}} + Q G_N^2 m_1 m_2 \hbar \log(-q^2) + Q' G_N^2 m_1 m_2$$

The Post-Newton and quantum corrections are *long range* contributions independent of the UV

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The one-loop UV divergence is a contact term of no interested to us

$$\delta V(r) = \delta^3(x) Q' G_N^2 m_1 m_2$$

Quantum gravity as an effective field theory

[John Donoghue] has explained that in an effective field theory treatment of quantum gravity one can evaluate long-range (infra-red) contributions and obtain reliable answers independent of the UV completion

These corrections depend only on the structure of the effective tree-level Lagrangian, the massless spectrum and the background

Any theory of quantum gravity should give the same result

Black-Hole entropy and AdS/CFT

[Ashoke Sen] has showed that the log correction to the entropy of non-extremal black holes can be computed in any quantum gravity theory

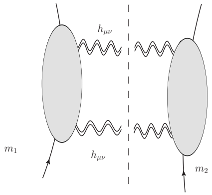
$$S = \frac{Area}{4\ell_p^2} + c \log\left(\frac{A}{\ell_p^2}\right) + \cdots$$

The coefficient c is universal because it only depends on the low-energy spectrum determined by the massless fields and their coupling to the background

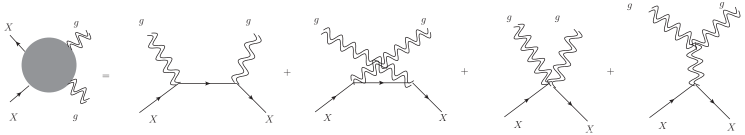
Any theory of quantum gravity should reproduce this coefficient

Making quantum gravity simple

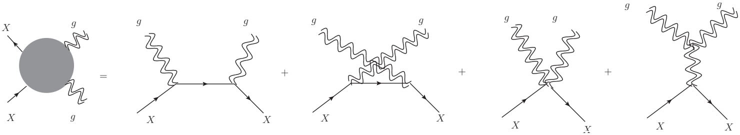
We are not interested in the full amplitude only the long range contributions matters



They are obtained by looking at the graviton cut and factorizing the amplitude on a product of *Gravitational Compton scattering*

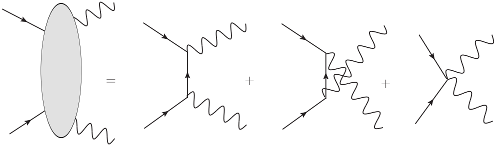


Gravitational compton scattering



The gravity Compton scattering as a product of two Yang-Mills amplitudes

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2} \mathcal{A}_s(1324)\mathcal{A}_0(1324)$$

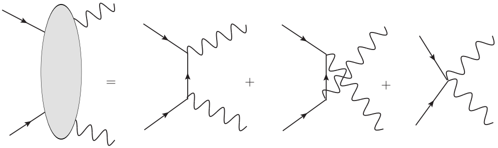


This relation is valid for massive matter external legs

[Holstein, Ross], [Bern, Carrasco, Johansson], [Kawai, Lewellen, Tye]

[Bjerrum-bohr, Damgaard, Vanhove], [Stieberger]; [Mafra, Schlotterer]

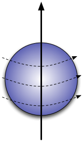
Gravity as square gauge theory



The remarkable property is that for this (color preserving) process the gauge theory amplitudes are the Abelian QED Compton amplitudes

[Holstein, Ross], [Bjerrum-Bohr, Donoghue, Vanhove]

A natural value for the Gyromagnetic ratio



The classical value of the g-factor for the electron is $g_0 = 2$ and Quantum mechanically $g = g_0 + \text{quantum corrections}$

$$g_{electron} = g_0 \left(1 + \frac{\alpha}{2\pi} + \cdots\right) = 2 \times 1.00115965$$

There was the question of the natural value of g_0 for spin S particle.

Belinfante conjectured that $g_0 = 1/S$ - but various arguments favored $g_0 = 2$ independently of the spin

A natural value for the Gyromagnetic ratio

Massive spin 1 couple to the photon by an anomalous Pauli interaction

$$\delta\mathcal{L} = -ie(g_0 - 1)F^{\mu\nu}\,\Big((W_\mu^+)^\dagger(W_\nu^+) - (W_\mu^-)^\dagger(W_\nu^-)\Big)$$

This leads to a piece of the amplitude that diverges for $m^2 \to 0$ if $g_0 \neq 2$

$$\delta\mathcal{A}_1 = \frac{(g_0-2)^2}{m^2}\,\left(\frac{n_s}{s} - \frac{n_t}{t}\right)$$

If $g_0 \neq 2$ [Weinberg; Porrati, Ferrara, Telegdi]

- Violation of unitarity for photon of energy $E \sim m$
- QED gets strongly coupled at energies $E \sim m/e$

A natural value for the Gyromagnetic ratio

The QED Compton amplitude is the 'square-root' of the gravity Compton scattering

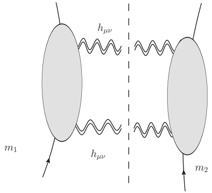
$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{p_1 \cdot k_1 \, p_1 \cdot k_2}{k_1 \cdot k_2} \mathcal{A}_s(1324) \mathcal{A}_0(1324)$$

For a massive spin 1 [Barry Holstein] noticed that gravity amplitude does not have any $1/m^2$ singularity and extracted the classical g-factor $g_0 = 2$

It is the two derivative nature of gravity that removes the $1/m^2$ for $m \to 0$

The relation *gravity* $\sim$ $(gauge)^2$ leads to $g_0 = 2$ for *all* values of the spin S

The one-loop amplitude



- The singlet cut gives a scalar box

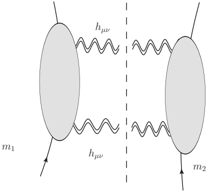
$$\mathfrak{M}|_{singlet\ cut} = \int \frac{d^{4-2\epsilon}\ell\,(m_1^2 m_2^2 s)^2}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i} = m_1^4 m_2^4 (I_4(s,t) + I_4(s,u))$$

- The non-singlet cut gives

$$\mathfrak{M}|_{non-singlet\ cut} = \int d^{4-2\epsilon}\ell \frac{\Re\mathrm{e}\left(\mathrm{tr}_-(\not{\ell}_1 \not{p}_1 \not{\ell}_2 \not{p}_2)\right)^4}{s^2 \ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i}$$

The numerators of the gravity amplitudes are the square of the one for a QED computation

The one-loop amplitude

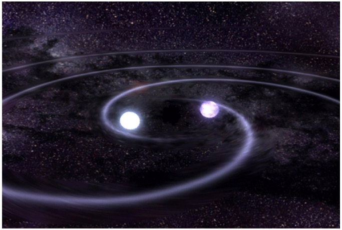


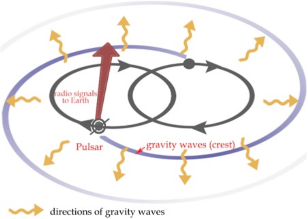
In the non-relativistic limit the $1/\sqrt{-q^2}$ and $\log(-q^2)$ coefficients are easily identified

Any terms like $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n\log(-q^2)$ are negligible

$$\mathfrak{M}^{non-rel}_{1-loop} = G_N^2 m_1 m_2 \left(\underbrace{6\pi}_{C} \frac{m_1+m_2}{\sqrt{-q^2}} \underbrace{-\frac{41}{5}}_{Q} \log(-q^2) \right)$$

Spin dependence





In the non-relativistic limit one can consider singlet, spin-orbit, quadrupoles, …

The coefficients C and Q have a spin-independent and a spin-orbit contribution

$$C,Q = C,Q^{S-I}\,\langle S_1|S_1\rangle\,\langle S_2|S_2\rangle + C,Q^{S-O}_{1,2}\,\langle S_1|S_1\rangle\,\vec{S}_2\cdot\frac{p_3\vec{\times}p_4}{m_2} + (1\leftrightarrow 2)$$

Universality of the result

Remarkably the coefficients are universal independent of the spin of the external states, a property noticed by [Holstein, Ross].

This is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems of [Low, Gell-Mann, Goldberger] and [Weinberg]

In the non-relativistic limit the QED Compton amplitudes take a simplified form given by

$$\mathcal{A}(X^s\gamma \to X^s\gamma) \simeq \langle S|S\rangle\, \mathcal{A}^{Compton} + \hat{\mathcal{A}}\,\vec{S}\cdot\frac{p_1\times p_2}{m}$$

For the Compton scattering

$$\mathcal{A}^{Compton} = \vec{\epsilon}_1\cdot\vec{\epsilon}_2^{\,*}\,\left(-\frac{e^2}{m} + \text{spin-orbit}\right) + i\vec{\sigma}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_2)\,\left(\frac{e^2g^2}{m^2}|k_1| + \text{spin-orbit}\right)$$

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This is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems of [Low, Gell-Mann, Goldberger] and [Weinberg]

In the non-relativistic limit the QED Compton amplitudes take a simplified form given by

$$\mathcal{A}(X^s\gamma \to X^s\gamma) \simeq \langle S|S\rangle\, \mathcal{A}^{Compton} + \hat{\mathcal{A}}\, \vec{S}\cdot \frac{p_1\times p_2}{m}$$

The KLT formula transports these theorems to gravity

$$\mathfrak{M}(X^s g \to X^s g) \simeq \langle S|S\rangle\, \mathfrak{M}(X^0 g \to X^0 g) + \hat{\mathfrak{M}}\, \vec{S}\cdot \frac{p_1\times p_2}{m}$$

In the cut this leads to universality of the result [Bjerrum-Bohr, Donoghue, Vanhove]

Outlook

Recent progress based on string theory techniques, on-shell unitarity, the double-copy formalism greatly simplifies perturbative gravity amplitudes computations

- The amplitudes relations discovered in the context of massless supergravity theories *extend* to the pure gravity case *with massive matter*
- The use of quantum gravity as an effective field theory facilitates the computation of universal contributions from the long-range corrections and the universality properties of coefficients in the effective potential

At the IPhT (CEA-Saclay) we organize
Amplitude 2014, a Claude Itzykson memorial conference
June 9 - 13, 2014