Can we detect a graviton?

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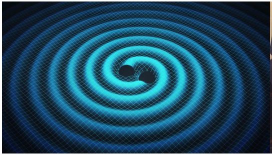
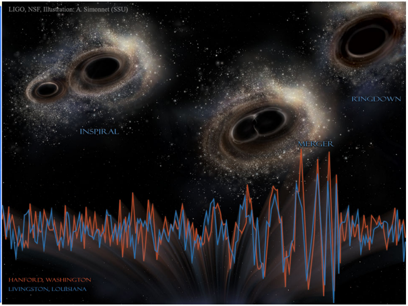
Gravitational waves

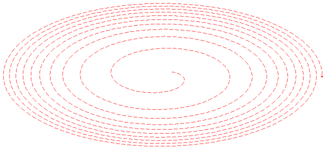


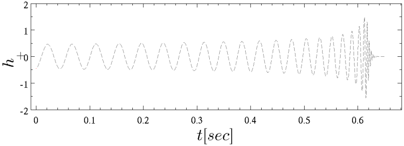
Septembre, the 14th 2015 LIGO detected gravitational waves emitted by the merging of a binary black hole system



Theoretical prediction

This detection is the result of an amazing theoretical works which predicted the waveform for various binary gravitational systems





This provides a direct proof of the reality of gravitational waves and the dynamical aspect of black hole

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$, and the final black hole mass is $62^{+4}_{-4}M_\odot$, with $3.0^{+0.5}_{-0.5}M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

Properties of the binary black hole merger GW150914

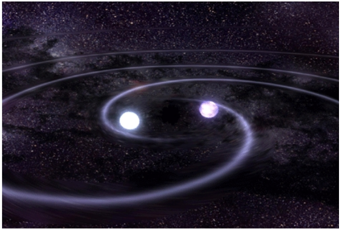
The LIGO Scientific Collaboration and The Virgo Collaboration

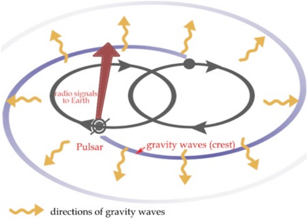
(compiled 12 February 2016)

On September 14, 2015, the Laser Interferometer Gravitational-wave Observatory (LIGO) detected a gravitational-wave transient (GW150914); we characterise the properties of the source and its parameters. The data around the time of the event were analysed coherently across the LIGO network using a suite of accurate waveform models that describe gravitational waves from a compact binary system in general relativity. GW150914 was produced by a nearly equal mass binary black hole of masses 36^{+5}_{-4} $M_\odot$ and 29^{+4}_{-4} $M_\odot$ (for each parameter we report the median value and the range of the 90% credible interval). The dimensionless spin magnitude of the more massive black hole is bound to be < 0.7 (at 90% probability). The luminosity distance to the source is 410^{+160}_{-180} Mpc, corresponding to a redshift $0.09^{+0.03}_{-0.04}$ assuming standard cosmology. The source location is constrained to an annulus section of 590 deg^2, primarily in the southern hemisphere. The binary merges into a black hole of mass 62^{+4}_{-4} $M_\odot$ and spin $0.67^{+0.05}_{-0.07}$. This black hole is significantly more massive than any other known in the stellar-mass regime.

This extends the validity regime of Einstein's generality to 10% of our observable Universe

Why is this important?

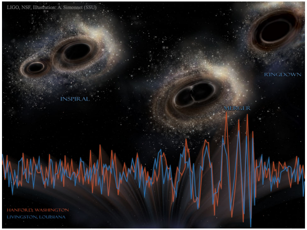




Einstein was doubtful about the physical reality of black hole

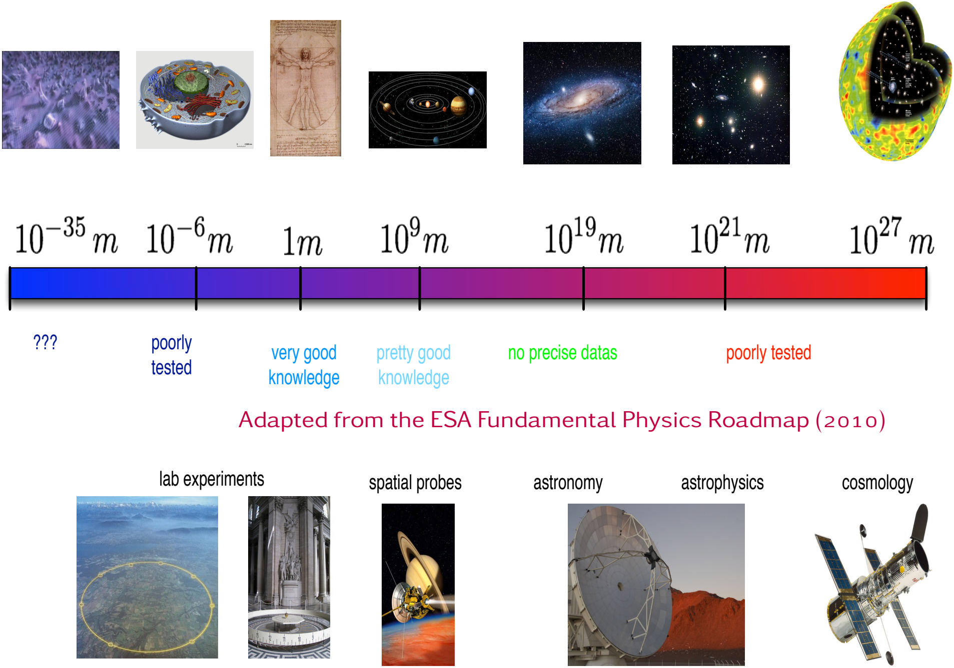
He was not convinced about the reality of gravitational waves

Why is this important?



The detection of GW150914 by LIGO opens a new window on our Universe

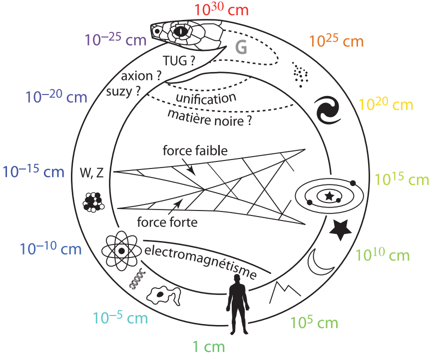
- For the first time detection and test of GR in the strong gravity coupling regime
- For the first time dynamics of Black hole (not just static object curving space-time)
- Constrain a number of theoretical mechanisms that modify GW propagation



Adapted from the ESA Fundamental Physics Roadmap (2010)

The importance of gravity

Gravity couples to all type of matter and energy



Gravity couples to any scale from very short to very large scales

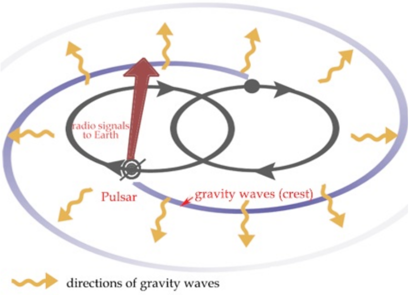
Many important questions about gravity

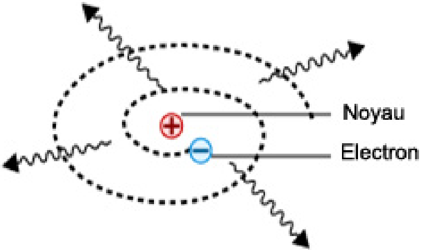
- Dark matter and dark energy problems
- black hole information paradox
- big bang and early cosmology
- do we need string theory? loop quantum gravity? or any other high energy theory?
- can a quantum field theory define correctly a quantum theory of gravity : c.f. maximal supergravity v.s. string theory
- Many more questions ...

The classical theory of gravity by Einstein is not enough

Why quantum gravity?

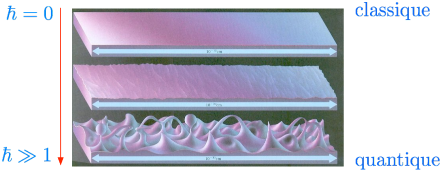
In 1916 Einstein advocated that "one should modify the new theory of gravity" to avoid the gravitational collapse of the atom by radiation of *gravitational waves*





He computed that it would take 10^{30} years (compared to the 10^{-11} second for EM radiation) but he also though that our Universe was eternal

Classical and quantum scales



- Classical scale: The Schwarzschild radius

$$r_S = \frac{2 G_N M}{c^2}$$

- Quantum scale: The Compton wave-length

$$\lambdabar = \frac{\hbar}{Mc}$$

- Dual with respect to the Planck length

$$r_S \lambdabar = 2\ell_P^2 = 2\frac{G_N \hbar}{c^3}$$

Classical and Quantum gravity contributions

How do these scale enters in practice?

Quantum ambiguity of the order of the Compton wave-length

$$\lambdabar = \frac{\hbar}{Mc}$$

$$\frac{1}{(r \pm \lambdabar)^2} \sim \frac{1}{r^2} \mp \frac{2\lambdabar}{r^3} + \cdots$$

Classical and Quantum gravity contributions

Starting from the post-Newtonian expansion of the gravitational potential

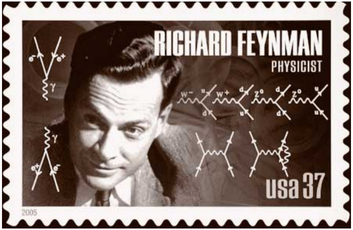
$$V(r) = \sum_{m \geqslant 0} v_{m,0} \left(\frac{r_S}{r} \right)^m$$

The quantum fluctuations would induce

$$V(r \pm \lambda\!\!\!^{-}) \simeq \sum_{m,n} v_{m,n} \frac{r_S^m \lambda\!\!\!^{-\,n}}{r^{m+n}} = \sum_{m,n} \hat{v}_{m,n} \frac{r_S^m \ell_P{}^{2n}}{r^{m+n}}$$

A quantum effect is naturally associated to classical GR contributions

Pertubative Quantum gravity



My subject is the quantum theory of gravity. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; ...(Feynman Jablonna, 1962)

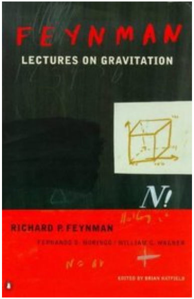
Perturbative Quantum gravity

- Starting from the Einstein-Hilbert action

$$\mathcal{S} = \frac{2}{32\pi G_N} \int d^4x \sqrt{-g} \left(\mathcal{R} + G_N g^{\mu\nu} T_{\mu\nu} \right)$$

- Perturbation around the flat space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{32\pi G_N}\, h_{\mu\nu}$$



- One can try to treat quantum gravity as an ordinary quantum field theory
- Propagating massless spin 2 particle : the graviton
- Similar to gauge theories with *huge gauge symmetry* from diffeomorphism invariance

Can we compute something meaningful?

Quantum gravity is still rather poorly understood although it expected to play a fundamental role in structure of our present universe.

Gravity is intrinsically non-linear, with a dimensional coupling constant and non-renormalisable

Can we extract meaningful physical quantities from a *quantum gravity* computation?

Can we compute something meaningful?

At one-loop there is a R^2 counter-term found by

$$S = \int d^4x |-g|^{\frac{1}{2}} \left[\Lambda + \frac{2}{32\pi G_N} \mathcal{R} + c_1 \mathcal{R}^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

The coefficients c_1, c_2 are unconstrained from the observation that $|c_i| < 10^{65}$

Since

$$\Lambda \simeq 10^{-123} M_{Pl}^4, \qquad M_{Pl} = 10^{19}\ GeV$$

Could it be that the real scale M_* is much lower than that $c_i \simeq (M_*/M_{Pl})^2$

What is the scale of observable quantum gravity effects?

Quantum gravity as an effective field theory

Steven Weinberg and John Donoghue have explained that one can evaluate some long-range infra-red contributions in any quantum gravity theory and obtain reliable answers

Some physical properties of quantum gravity are *universal* being independent of the UV completion

The infra-red contributions depend only the structure the low-energy fields and the classical background

Quantum gravity as an effective field theory

The effective field theory is an *effective* limit of a more complicated theory that still capture non trivial quantum effects

We assume

- The quantum field theory and its quantisation
- the set of degrees of freedom at a given scale (massless graviton)
- the observed symmetries (diffeomorphism invariance)

Why is this computation meaningful?

The Post-Newton and quantum corrections are *long range* contributions independent of the UV

This is why in the infrared there are some physical properties of quantum gravity are *universal* being independent of the UV completion

These corrections depend only on the structure of the effective tree-level Lagrangian, the massless spectrum and the background

Why is this computation meaningful?

The classical result is used to check the validity of the approach as we should reproduce General relativity. The quantum corrections are new contributions.

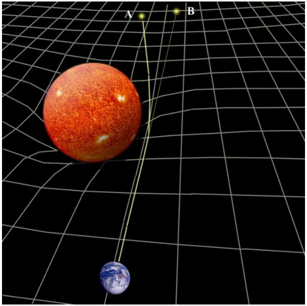
On the other side the techniques we hope that the techniques we use will be useful to compute efficiently the higher order Post-Newtonian contributions needed for precision gravitational wave physics.

Any theory of quantum gravity should give the same result

Physics of the effective field theory approach

With the effective field theory approach we can compute

- the classical (post-Newtonian) contributions to gravitationally bounded system
- universal long range quantum corrections extending the predictions of general relativity



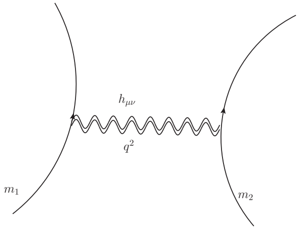
Perturbative technics

Classical Newton's potential is obtained in the non-relativistic limit of a 1-graviton exchange

$$\mathfrak{M}(\vec{q}) = \frac{G_N m_1 m_2}{\vec{q}^2}$$

The potential is

$$V(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{4m_1 m_2} \mathfrak{M}(\vec{q})\, e^{i\vec{q}\cdot\vec{r}} = -\frac{G_N m_1 m_2}{r}$$



Classical and Quantum contributions

We want to derive the corrections to the non-relativistic Newton's potential

$$V(r) = -\frac{G_N m_1 m_2}{r} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + Q\,\frac{G_N^2 m_1 m_2 \hbar}{r^3} + \cdots$$

Take a Fourier transform

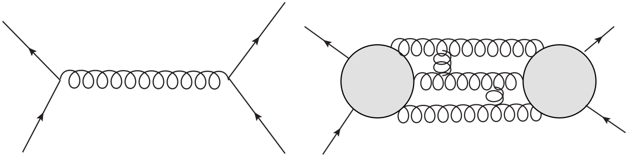
$$\mathfrak{M}(q) = \int d^3\vec{x}\, e^{i\vec{q}\cdot\vec{x}} V(r)$$

to get the amplitude

$$\mathfrak{M}(q) = \frac{G_N m_1 m_2}{q^2} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{\sqrt{-q^2}} + Q\, G_N^2 m_1 m_2 \hbar \log(-q^2) + \cdots$$

Classical and Quantum contributions

We will be considering the pure gravitational interaction between massive and massless matter of various spin



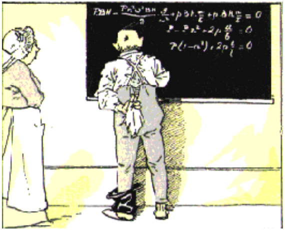
$$\mathfrak{M} = \frac{1}{\hbar}\,\mathfrak{M}^{\text{tree}} + \hbar^0\,\mathfrak{M}^{1-\text{loop}} + \cdots .$$

The higher order corrections arise from graviton loops

$$\mathfrak{M}(q) = \frac{G_N m_1 m_2}{q^2} + C\,\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{\sqrt{-q^2}} + Q\, G_N^2 m_1 m_2 \hbar \log(-q^2)$$

The UV divergences are short distance contact term of no interested to us

Classical physics from loops



We want to extract the classical and quantum correction from a one-loop computation

- Quantum corrections of order $\hbar^n$ requires an n-loop amplitude computation
- A loop amplitude can give rise to a classical, ie of order $\hbar^0 = 1$, contribution

Classical physics from loops

What is reason for the appearance of classical contribution at loop order?

At each *vertex* we have a power of $\hbar^{-1}$ from

$$e^{\frac{i}{\hbar}\int d^4x \mathcal{L}_{int}(x)}$$

For each propagator we get a power of $\hbar$ from

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int d^4k \, \frac{i\hbar}{k^2 - \frac{m^2c^2}{\hbar^2} + i\varepsilon} \, e^{ik\cdot(x-y)}$$

A graph with V vertices, I propagators and L loops has

$$\hbar^{-V+I} = \hbar^{L-1}$$

Classical physics from loops

What is reason for the appearance of classical contribution at loop order?

At each *vertex* we have a power of $\hbar^{-1}$ from

$$e^{\frac{i}{\hbar}\int d^4x \mathcal{L}_{int}(x)}$$

For each propagator we get a power of $\hbar$ from

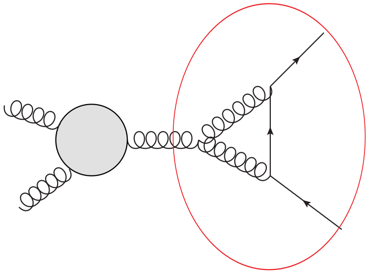
$$\langle 0|\phi(x)\phi(y)|0\rangle = \int d^4k \, \frac{i\hbar}{k^2 - \frac{m^2c^2}{\hbar^2} + i\varepsilon} \, e^{ik\cdot(x-y)}$$

But in a non-relativistic limit *mass depend terms* can arise with no $\hbar$

$$\hbar \times \frac{m}{\hbar\sqrt{-q^2}} = \frac{m}{\sqrt{-q^2}}$$

Classical physics from loops

Let's consider the one-loop contribution for a say a massive scalar of mass m



The triangle contribution with a massive leg $p_1^2 = p_2^2 = m^2$ reads

$$\int \frac{d^4\ell}{(\ell+p_1)^2(\ell^2-\frac{m^2c^2}{\hbar^2})(\ell-p_2)^2}\Bigg|_{\text{finite part}} \sim \frac{1}{m^2}\left(\log(s)+\frac{\pi^2 mc}{\hbar\sqrt{s}}\right)$$

Classical physics from loops

The $1/\hbar$ term at one-loop contributes to the *same* order as the classical tree term

$$\mathfrak{M} = \frac{1}{\hbar}\left(\frac{G_N (m_1 m_2)^2}{\vec{q}^2} + \frac{G_N^2 (m_1 m_2)^2 (m_1 + m_2)}{|\vec{q}|} + \cdots\right) + \hbar^0 G_N^2 O(\log(\vec{q}^2)) + \cdots \tag{1}$$

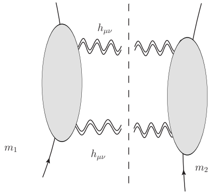
For the scattering between a massive matter of mass m and massless matter of energy E one gets

$$\mathfrak{M} \sim \frac{1}{\hbar}\left(G_N \frac{(mE)^2}{\vec{q}^2} + G_N^2 \frac{m^3 E^2}{|\vec{q}|}\right) + \hbar\, G_N^2 O\left(\log(\vec{q}^2), \log^2(\vec{q}^2)\right)$$

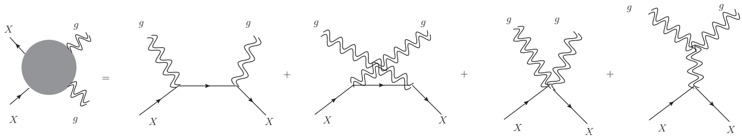
The mechanisms generalises to higher loop-order amplitudes to leads to the higher order post-Newtonian corrections

Making quantum gravity simple

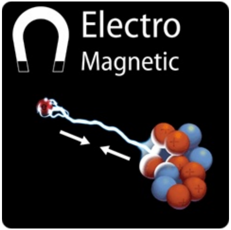
We are not interested in the full amplitude only the long range contributions matters

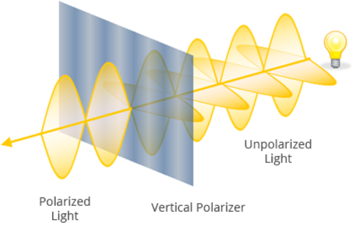


They are obtained by looking at the graviton cut and factorizing the amplitude on a product of *Gravitational Compton scattering*



Quantun of light : the photon

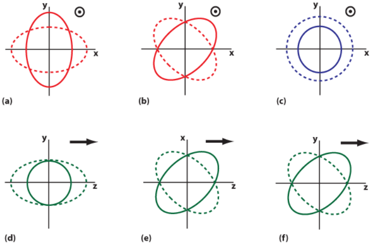




Photon : massless, spin 1, quantum of electromagnetic waves

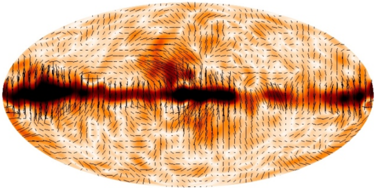
$$\gamma: \qquad \epsilon_\mu^+, \qquad \epsilon_\mu^-, \qquad \text{mass} = 0$$

Quantum of space-time: the graviton

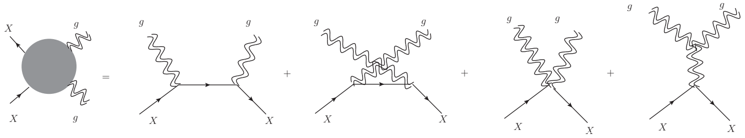


Graviton : massless, spin 2, (hypothetical) quantum of space-time fluctuations

$$h: \qquad \epsilon^{++}_{\mu\nu}, \qquad \epsilon^{--}_{\mu\nu}, \qquad \text{mass} = 0$$

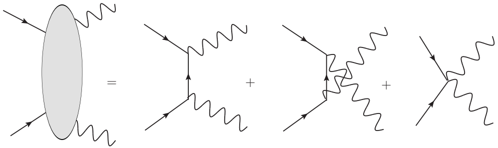


Gravitational compton scattering



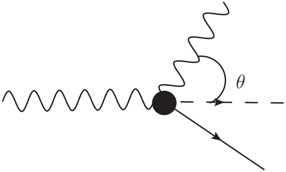
The gravity Compton scattering as a product of two Yang-Mills amplitudes

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2} \mathcal{A}_s(1324)\mathcal{A}_0(1324)$$



This relation is valid for massive matter external legs

Compton scattering

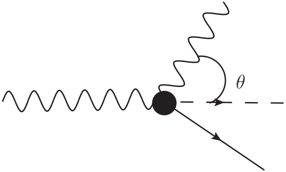


The Compton scattering has the universal low-energy limit

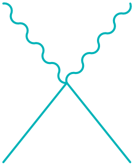
$$\left.\frac{d\sigma^{\text{Comp}}_{lab,S}}{d\Omega}\right|^{NR} = \frac{\alpha^2}{2m^2}\left[\left(\cos^4\frac{\theta}{2}+\sin^4\frac{\theta}{2}\right)\left(1+\mathcal{O}\left(\frac{\omega_i}{m}\right)\right)\right]$$

This is a consequence of the well-known low-energy theorems in QED

Compton scattering

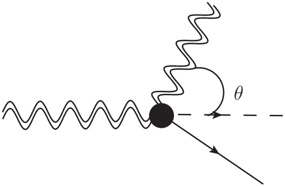


It's small angle limit is dominated by the contact interaction



$$\lim_{\theta \to 0} \frac{d\sigma^{\mathrm{Comp}}_{lab,S}}{d\Omega} = \frac{e^2}{8\pi m^2},$$

Gravitational Compton scattering

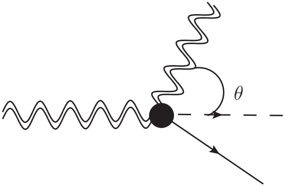


The low-energy limit of the gravitational Compton scattering of gravitons on a massive target

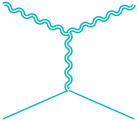
$$\frac{d\sigma^{\mathrm{g-Comp}}_{lab,S}}{d\Omega} = G^2 m^2 \left[\mathrm{ctn}^4 \frac{\theta}{2} \cos^4 \frac{\theta}{2} + \sin^4 \frac{\theta}{2} + \mathcal{O}\left(\frac{\omega_i}{m}\right)\right]$$

This is a consequence of the fact that the gravitational Compton scattering is the product of two Compton scattering amplitudes and the low-energy theorems in QED

Gravitational Compton scattering



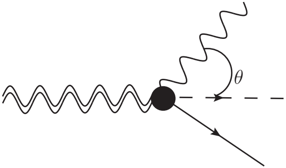
At small-angle, we have Rutherford behavior of a $\frac{1}{r}$ long-range potential.



$$\lim_{\theta \to 0} \frac{d\sigma_{lab,S}^{\mathrm{g-Comp}}}{d\Omega} = \frac{16 G^2 m^2}{\theta^4}.$$

The interaction is dominated by the graviton pole

Graviton photoproduction scattering

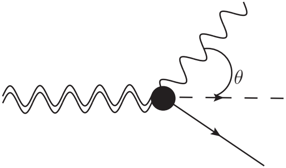


The double copy relation expresses the amplitude as

$$\mathfrak{M}(X^s g \to X^s \gamma) = \frac{\kappa}{2e} \frac{p_f \cdot F_f \cdot p_i}{k_i \cdot k_f} \times \mathcal{A}_s(1234)$$

the product of the Compton scattering amplitudes on target of spin S times a form factor

Graviton photoproduction scattering

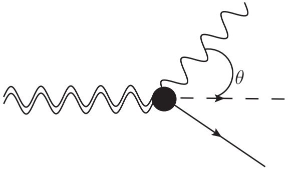


The low-energy limit of the graviton photoproduction cross-section

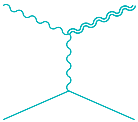
$$\frac{d\sigma^{\text{photo}}_{lab,S}}{d\Omega} \xrightarrow[\omega \to 0]{} G\alpha \cos^2\frac{\theta}{2}\left(\mathrm{ctn}^2\frac{\theta}{2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right).$$

This is again independent of the spin of the target as a consequence of the QED low-energy theorems

Graviton photoproduction scattering



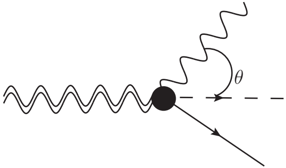
The small angle limit has the behavior of an effective $1/r^2$ potential



$$\lim_{\theta\to 0} \frac{d\sigma^{\rm photo}_{lab,S}}{d\Omega} = \frac{4G\alpha}{\theta^2}\,,$$

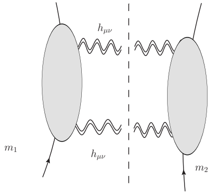
This is dominated by the photon pole

Graviton photoproduction scattering



- Very weak source of gravitational waves from nuclear reactions in the stars
- if detectable this would lead direct detection of a graviton by illumination

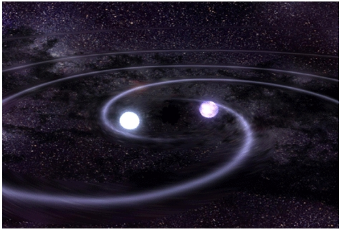
The one-loop amplitude

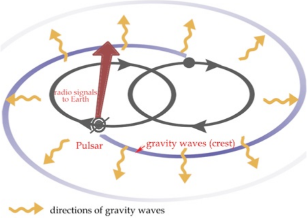


The numerators of the gravity amplitudes are the square of the one for a QED computation

$$\mathfrak{M}_{1-loop}^{non-rel} = G_N^2 m_1 m_2 \left(\underbrace{6\pi}_{C} \frac{m_1 + m_2}{\sqrt{-q^2}} \underbrace{-\frac{41}{5}}_{Q} \log(-q^2) \right)$$

Spin dependence





In the non-relativistic limit one can consider singlet, spin-orbit, quadrupoles, ...

$$C, Q = C, Q^{S-I} \langle S_1|S_1\rangle \langle S_2|S_2\rangle + C, Q^{S-O}_{1,2} \langle S_1|S_1\rangle \vec{S}_2 \cdot \frac{p_3 \vec{\times} p_4}{m_2} + (1 \leftrightarrow 2)$$

The coefficients C and Q have a spin-independent and a spin-orbit contribution

Universality of the result

Remarkably the coefficients are universal independent of the spin of the external states

This is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems in QED

In the non-relativistic limit the QED Compton amplitudes take a simplified form given by

$$\mathcal{A}(X^s\gamma \to X^s\gamma) \simeq \langle S|S\rangle \mathcal{A}^{Compton} + \hat{\mathcal{A}}\,\vec{S}\cdot\frac{p_1\times p_2}{m}$$

For the Compton scattering

$$\mathcal{A}^{Compton} = \vec{\epsilon}_1\cdot\vec{\epsilon}_2^{\,*}\left(-\frac{e^2}{m} + \text{spin-orbit}\right) + i\vec{\sigma}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_2)\left(\frac{e^2g^2}{m^2}|k_1| + \text{spin-orbit}\right)$$

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The double copy formula implies for gravity

$$\mathfrak{M}(X^s g \to X^s g) \simeq \langle S|S\rangle \,\mathfrak{M}(X^0 g \to X^0 g) + \hat{\mathfrak{M}}\,\vec{S}\cdot\frac{p_1\times p_2}{m}$$

In the cut this leads to universality of the result

Universality of the result

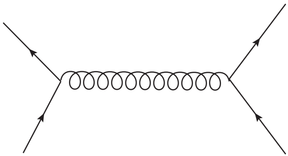
For the classical contribution

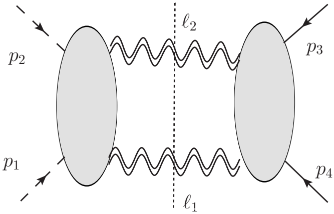
- This is totally what one expects from the equivalence principle and the multipole expansion of the gravitational interaction between massive states

For the quantum contributions

- The long range quantum correction involves low-energy gravity degrees of freedom and is independent of any microscopic high-energy model dependent contributions

The one-loop amplitude for massless particles





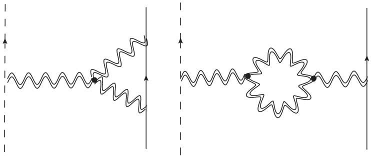
For photon scattering only the amplitudes with helicity $(++)$ and $(--)$ are non-vanishing.

Therefore there is no birefringence effects to contrary to case with electrons loops contributing to the interaction

The bending angle for a massless particle

$$\theta_S \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8bu^S + 9 - 48\log\frac{b}{2b_0}}{\pi}\,\frac{G^2\hbar M}{b^3}\,.$$

- The classical contribution including the 1rst Post-Newtonian correction is correctly reproduced
- The quantum corrections are new: not only from a quantum corrected metric



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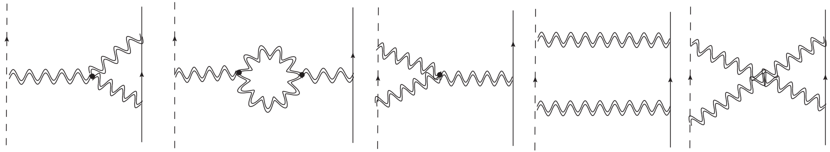


The difference between the bending angle for a massless photon and massless scalar is independent of the IR regulator

$$\theta_\gamma - \theta_\varphi = \frac{8(bu^\gamma - bu^\varphi)}{\pi}\frac{G^2\hbar M}{b^3}\,.$$

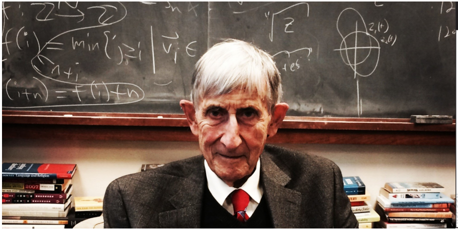
The physics of quantum gravity

Recent progress based on string theory techniques, on-shell unitarity, the double-copy formalism greatly simplifies perturbative gravity amplitudes computations

The amplitudes relations discovered in the context of massless supergravity theories *extend* to the pure gravity case *with massive matter*

The use of quantum gravity as an effective field theory facilitates the computation of universal contributions from the long-range corrections and the universality properties of coefficients in the effective potential

Can we detect a graviton?



In his Poincaré Prize lecture Freeman Dyson question the possibly of detecting single gravitons

We have explained that one can obtain non trivial quantum effects without having to detect graviton directly

What would we learn if detected a graviton? We can quantize phonons and other quasiparticles even though they do not exist as fundamental particles at all scales

The main question is what experiment will provide an hint of the quantum nature of space time and its symmetries