If it's not $\mathcal{DARK}$*,*
it doesn't $\mathcal{MATTER}$

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In this paper I expose the problem of the missing matter in the universe. I review some general methods of measurement of the mass of the universe, and conclude that there is some sign of missing matter in the universe. After I expose the methods to detect a possible candidate for this matter, the neutralino, one of the particles predicted by the supersymmetrical theory. I explain how to compute the detection rate of the neutrinos produced by the annihilation of this candidate in the Sun or in the Earth.

Subject headings: dark matter—supersymmetrical theory: neutralinos—solar system: Sun & Earth--detection: neutrinos & muons detection.

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About this report :
This report contains three appendixes, their reading is not necessary, but I advice the reading of the appendix II. So I count the appendix II in the 25 pages allowed for the report, but not the appendix I and III.

Word processing was carried with the typesetting system TEX on a Sun SPARC station. The font is the computer roman 11 points with a baseline skip of 16 points. The paper including the front page, the table of contents, the page of references but without the tables and the figures is 25 pages long.

Notation: When a new quantity is defined I will use the symbol $\widehat{=}$

[...], *une présentation originale met parfois mieux en valeur le travail effectué ;* [...]
[...], *il comprend instantanément à condition de lui expliquer.*

Christophe Dupraz in memento de stage à l'étranger—1993

1 – INTRODUCTION

In 1844, Friedrich Whilhem Bessel, after a decade of positional measurement of Sirius, concluded of the existence of a non-luminous companion. This companion, known as the white Dwarf Sirius B, was first observed in 1962 by Alvan G. Clark with an $18\frac{1}{2}$" objective.

Later, Oort inferred from his measurements of the numbers and the velocities of stars near the Sun that the visible light represents only 30–50% of the gravitational matter (this is the so-called Oort limit).

In 1929 Hubble deduced from his observations that the universe is expanding "homogeneously". He found that an astrophysical object at a distance of r goes away from us with a radial velocity $v = \mathrm{H}\, r$, where $\mathrm{H}= \dot{\mathrm{R}}(t)/\mathrm{R}(t)$ is the Hubble constant. Another important parameter is $q = -\ddot{R}/\dot{R}\, H$ the deceleration parameter, R represents the scale factor of the universe and the derivative is calculated with respect to the time t (*cf.* Appendix III). The value for H_0 calculated by Hubble was $\mathrm{H}_0 \sim 519\,\mathrm{km/s\ Mpc}$.

In 1984 Ostriker, Yahil & Peebles and Einasto, Kraasik & Saar deduced from their measurements of the mass M for elliptical and spiral galaxies as a function of the radius r, that $\mathrm{M}(r)$ grows linearly with the radius, $\mathrm{M}(r) \propto r$. They found that for radii varying up to 100 kpc, M increases up to $10^{12}\ \mathrm{M}_\odot$. [1]

Nowadays a large amount of astrophysicists is convinced that we do not detect at least 90% of the matter in the universe. That is the so-called problem of the dark matter. The dark matter includes all the non-luminous matter of the universe, *i.e.*, all the matter that we cannot detect.

Although many physicists are convinced of the existence of the dark matter , there is no real proof of it. We must indicate that there exist some system where the gravitationnal mass is equal to the luminous mass which not need some dark matter.

[1] In astrophysics the distances are measured in parsec: 1 pc $= 3.09 \times 10^{18}$cm $= 3.26$ light years, and the masses in solar masses $\mathrm{M}_\odot = 1.99 \times 10^{33}$ g.

2 – THE MASS OF THE UNIVERSE

I will not give any references in this chapter since all the information is taken from the article writen by V. Trimble (1990).

2.1 – General methods

The determination of the mass density of the universe ρ_0 (I will use the subscript 0 for the present value of any cosmological quantity) is a real challenge, and nowadays many physicists try to cope with it.

In general cosmologists quantify the mass of the universe by the mass-energy density divided by the critical density, $\rho_c \,\hat{=}\, 3\mathrm{H}_0^2/8\pi\mathrm{G}$, needed to close the universe

$$\Omega_0 \,\hat{=}\, \rho/\rho_c.$$

Because of the uncertainties on H_0, $40\,\mathrm{km\ s^{-1}\ Mpc^{-1}} \lesssim \mathrm{H}_0 \lesssim 100\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$, the Hubble parameter in unit of $100\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$, h, is often introduced ; thus $\rho_c = 2.76 \times 10^{11}\ h^2\ \mathrm{M_\odot\,Mpc^{-3}}$.

The simplest method to determine the mass density, which gives some good results, is to calculate the mean mass density, averaged over thousands of Mpc,[2] $\langle\rho_0\rangle = n_{Gal}\,\langle \mathrm{M}_{Gal}\rangle$ where n_{Gal} is the number density of the galaxy and $\langle \mathrm{M}_{Gal}\rangle$ the average mass per galaxy.

Three different methods can be used to measure the mass of a galaxy or a region of the outer space. For instance for single galaxies we can use the velocity dispersion of stars or the rotation curves (the relation between the distance of the observer and a test body and its velocity perpendicular to the line of sight) of gas—including X-ray-emitting gas—making up the galaxy itself. Or the velocities test particles like globular clusters and satellite galaxies. All these methods are in good agreement (in fact we must take the same value for the free parameters of the theory: $\mathrm{H}_0, \ldots$).

In most cases, if we assume that the galaxy is spherically or radially symmetric, the only physics needed is given by the fundamental relation of dynamics for a body in a gravitational potential created by a sphere—there are only minor differences if the source is elliptic, or a thin disc.

So the mass interior to a radius r is given by

$$\mathrm{GM}(r) = v^2 r.$$

The physicists expected that the velocity v grows with the radius and after a radius R_0 decreases as $v \propto r^{-1/2}$ showing that the mass of the object is contained in the radius R_0. By taking the radius within which the galaxy emits most of its light one finds

$$\Omega_{Lum} \sim 0.01 \quad \text{or less.}$$

[2] For comparison the radius of the observable volume of the universe is about $H_0^{-1} \simeq 3000h^{-1}$ Mpc $\sim 10^{28}h^{-1}$ cm.

But if we take a radius r greater than this luminous radius—by observing the rare stars, the emission of the 21 cm line from neutral H, the gas clouds—we find

$$\mathrm{M}(r) \propto r,$$

Which shows that there is more mass out of the luminous radius of the galaxy.

One can use the peculiar velocities (the velocities of a body after we have substacted the velocity due to then expansion of the universe) of some density enhancement such that the Virgo cluster (which represents a relative variation of $\delta n_{Gal}/n_{Gal} \sim few$ at $20\,\mathrm{Mpc}$ from us) to determine $\langle M_{Gal} \rangle$ for larger structures. This method gives $\Omega_0 \sim 0.1$–0.2.

And on more larger scales one can use, under the assumption that the system is bounded and well relaxed, the virial theorem $\mathrm{MG} = 2\left\langle v^2 \right\rangle / \left\langle r^{-1} \right\rangle$ to relate the kinetic energy relative to the Hubble flow to the gravitation potential determined by the mass density.

Oort pioneered the analysis of stellar velocity and distribution orthogonal to the galactic plane. His results are consistent with the more recent from Bahcall, who found for a sphere of radius $r < 0.7$ kpc $\rho_{dark}/\rho_{Lum} = 0.5$–$1.5$, that is to say $\Omega_0 \sim 0.02$ or a density near $0.1\ \mathrm{M}_\odot\,\mathrm{pc}^{-3}$ (Trimble 1990).

Table 1 gives a summary of the values of Ω_0 calculated by the previous methods on different scales.

2.2 – Caveats

We must keep in mind that these values relie on theoretical or experimental parameters very difficult to evaluate.

First theoretical uncertainties may come from:

- The Hubble parameter H_0 is difficult to determine and its value has been greatly modified since the first determination of Hubble $\mathrm{H}_0 \approx 500\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$ because of an over-estimation of the luminosity of the stars used as reference for distances.
- Because of uncertainties in the models of evolution of the stars, metallicity, and stellar brightness, the distances may be under-estimated and the local mass over-estimated. Because if for a given velocity the star is farther from us than thought, the gravitational mass attracting it is less than the mass calculated.
- For the halo of our galaxy there is the problem of having a sufficient number of objects whose distances and velocities can be measured, but it seems that the total mass of the Milky Way $\mathrm{M}_{MilkyWay} \lesssim 10^{12}\ \mathrm{M}_\odot$ and is contained in a radius[3] of *several* 10 Kpc.
- The application of the cosmic virial theorem would be wrong if the rich galaxy clusters are not yet relaxed or if there is some interaction with a subsystem or a dynamics dominated by a central massive core.

[3] The distance of the solar system from the center of the galaxy is 8.5 Kpc

Observational uncertainties:

- The test stars in the neighborhood of the Sun may be brighter than expected, and the gravitational mass might be overestimated.
- The luminosity at large radius may be underestimated because of the subtracted sky background brightness being overestimated. This would give some errors in the rotation curves of the Milky Way and other spiral Galaxies.

2.3 – Conclusion

We have seen that we must be aware of the uncertainties of the measured values for Ω_0. But it is safe to conclude that:

- For our galaxy in a radius of *several* 10 Kpc there is as much mass in a spheroidal (mostly dark) halo as within the luminous disc.
- And that for r larger than 10 Kpc there is a least 2, probably 3–10, times much matter.
- The non-luminous matter in the whole universe dominates the matter by a factor 10.
- The amount of matter that clusters in the halo of the galaxies on scales of 10 to 30 Mpc contributes for 0.2 of critical density.
- The dynamical measurement does not preclude a less clustered, or even smooth, component more massive.

3 – THEORETICAL EXPECTATIONS FOR Ω

The value $\Omega = 1$ is enhanced by the cosmology of Friedmann-Robertson-Walker (which just assume the homogeneity and isotropy of the universe, *cf.* Appendix III) and by the theoretical model for the evolution of the very early universe called Inflation. The inflation was a period of exponential expansion triggered by high-temperature phase transitions occurring when the vacuum energy is dominating, which guarantees that widely separated parts of the universe, *i.e.*, causally not connected, were in communication and can have now the same density and temperature.

$\Omega_0 = 1$ is the only fixed, but very unstable, value for Ω_0 in the Friedmann-Robertson-Walker's cosmology. [4] An extreme fine tuning of H and T, one part in 10^{55}, is needed for $\Omega = 1$ (Guth 1981).
Ω_0 less than 1 corresponds to an universe in expansion, and Ω_0 greater than 1 to a close universe. In both cases Ω_0 varies very rapidly with the time. Since it seems reasonable that there is

[4] in fact $\Omega \equiv 0$ is a stable fixed value but it means that there is nothing in the universe.

nothing particuliar with our epoch, $\Omega_0 = 1$ is grandly preferred. In the frame of the Friedmann-Robertson-Walker's cosmology Ω verify the equation (*cf.* the appendix III)

$$\frac{\Omega - 1}{\Omega} = \frac{3kR}{8\pi G\rho R^3} \tag{1}$$

where R is the scale factor of the universe and k a parameter of the theory (*cf.* Appendix III). This equation shows that we always have $(\Omega - 1)/\Omega \propto k$. As I said in the Appendix III, if the parameter of Friedmann-Robertson-Walker's cosmology k is greater than 0 the universe is closed, the universe is an unclosed and flat for $k = 0$ and is an unclosed and unbound for k negative (generally the coordinates are rescaled in a way that k takes one of the three values $\{+1, 0, -1\}$). As the value $\Omega = 1$ is exact only if $k = 0$, it would be a very strange matter of fact that our universe had precisely $k = 0$, but that is believed by many physicists.

This equation gives that during the nucleosynthesis epoch Ω was suprisingly very near 1 (Guth 1981): $|\Omega - 1| \lesssim 10^{-15}$ for $T \sim 1\,\text{Mev}$; at the Grand Unification scale $T \sim 10^{14}\,\text{GeV}$ the difference may be: $|\Omega - 1| \lesssim 10^{-49}$; at the Planck scale: $T \sim 10^{15}\,\text{GeV}$ $|\Omega - 1| \lesssim 10^{-59}$. Some people said that there was no problem since in any Friedmann-Robertson-Walker's cosmology Ω goes to 1 when the time goes to 0 (see appendix III for a definition of the time $t = 0$). When the time goes to 0, the "radius" of the universe $R(t)$ goes to 0 too therefore the right-hand-side of the equation (1)vanish (The quantity ρR^4 is a constant in the Radiation domination epoch, *cf.* appendix III). But as Guth stressed in his paper (Guth 1981) the Planck scale is not a singular scale particularly near 0 but an epoch like the other, and there is a real problem.

In theory of cosmic inflation the curvature of the universe vanishes $\Omega - \Omega_\Lambda - 1 = 0$, where Λ is the cosmological constant and $\Omega_\Lambda = \Lambda/3\text{H}^2$ (Klapdor *et al.* 1986). Theories and experimentations give Λ of the order of $4.7\text{–}19 \times 10^{-57}\,\text{cm}^{-2}$. Such a low value prompts many physicists to take its value equal to 0. We will see in section 4.5 that a non-vanishing Λ can be used to solve partly the problem of the dark matter. Hereafter, except in section 4.5, I suppose $\Lambda \equiv 0$. [5]

Observations provide some arguments for $\Omega_0 = 1$. If one takes $\Omega_0 < 1$, there will be some problem in forming the galaxies and the large scale structures without introducing larger homogeneities than measured by COBE[6] in the $2.735 \pm 0.060\,\text{K}$ cosmic micro-wave background (CMBR) (Roszkowski 1992). The limit on $\delta T/T$ is about $\delta T/T \lesssim few \times 10^{-5}$ on angular scales of 10 arcs seconds to 180°, which are the size of the galaxies at the epoch of the decoupling between matter and radiation, that is at an age of $180,000(\Omega_0 h^2)^{-1/2}\,\text{yr} \sim 200,000\,\text{yr}$. $\Omega_0 = 1$ gives too big anisotropies but $\Omega_0 < 1$ is worse. Some non-baryonic dark matter that does not interact with radiation at $T \lesssim 1\,\text{GeV}$ can give $\Omega_0 = 1$ without introducing big anisotropies. We must emphasis

[5] Einstein said that this cosmological constant was "the biggest blunder of" his life.

[6] COBE is an acronym for COsmic Background Explorer. This satellite was launched by NASA in November 1989 to an orbit at 900 km above the Earth's surface. It carries three major detectors: the Far InfraRed Absolute Spectrophotometer which scans a wide range of CMBR frequencies, the Diffuse InfraRed Background Experiment which searches for radiation due to galactic evolution and the Differential Micro-wave Radiometer which has been designed to perform a complete angular mapping of the CMBR.

that angular resolution of COBE is 7° and the angle under which we see the anisotropies due to the galaxies, gravitational inhomogeneities or particle physics interactions are of size 1°, sometimes lower. The values of the inhomogeneities at such sizes are only extrapolated from the datas of COBE and are submitted to caution.

If someone believes that $\Omega_0 = 1$, as the luminous matter contributes only at most to $\Omega_{Lum} = 0.2$, he may ask what is the composition of the other 4/5 of the mass density of the universe.

4 – THE DARK MATTER CANDIDATES

For this chapter the best reference is the book from Kolb and Tuner (Kolb & Turner 1988).

If there exists some non-luminous dark matter on many scales, what can be its composition ? The constituents must have a sufficiently small speed to stay in the galaxies but also have a velocity big enough to be less clustered than the superclusters.

Now I review some possible dark matter candidates, and explain the advantages and drawbacks to consider them as dark matter candidates.

4.1 – Baryonic dark matter

Baryonic matter (matter governed by the strong interactions like the nucleus of the atoms and the ions) seems to be a good candidate since we know the existence of some baryonic non-luminous astrophysical objets, like Jupiter, the white Dwarfs, the neutron stars, the brown Dwarfs (not yet detected because very cool and faint), the black holes ...These Massive Astrophysical Compact Halo Objects (MACHOs) with a mass between 10^{-7} and 0.08 $M_\odot$ are balls of Hydrogen and Helium (if the mass of the object is lower than 10^{-7} $M_\odot$ the gas evaporates and if its mass is greater than 0.08 $M_\odot$ the object starts to burn) (Derujula *et al.* 1992).

Three groups are trying to detect them by micro-lensing: the american group composed by the Center for Particle Astrophysics ; a French collaboration ; and a collaboration between the Polish institution, the Carnegie Institute and Bohdan Paczynski who is the inventor of the idea. If there is a MACHO on the line of sight between us and an observed star (typically in the Magellanic Clouds) the rays are bended and a ring is observed ; if the alignment is not good two images will be observed. In fact the angle between the two images is less than the resolution if the detector, thus the groups are looking for stars looking brighter periodically during a definite time, depending on the mass of the MACHOs. But the theory of the Hot Big Bang and the abundances of the light nucleides—H, D, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li—impose very stringent limits on the fractions of baryons

$$0.015 \le \Omega_{\rm B} h^2 \le 0.026.$$

With $0.4 \leq h$, $\Omega_B \lesssim 0.16$. Moreover theories with $\Omega_B \sim 1$ give not enough D and too much ^{4}He and ^{7}Li.

So there must be some non-baryonic dark matter in the universe.

4.2 – Non-baryonic dark matter

Hot and Cold dark matter

There are two kinds of dark matter: the Hot dark matter composed by relativistic particles like neutrinos, anti-neutrinos, electrons, muons and the Cold dark matter composed by non-relativistics elements better known under the acronym WIMPs for Weakly Interacting Massive Particles like the super-symmetric partner (*cf.* Table 2).

Formation of super-clusters and large scale structures are grandly favored in an universe dominated by some 10–100 eV Hot dark matter. Some models allows the dark matter to be composed only by one baryon species, but in a theory of Hot dark matter there is too big velocities dispersions on small scales and the galaxies are formed too late.

In an universe dominated by particle of mass of order MeV or GeV, the galaxies and the smaller structures are greatly favored to the detriment of the larger scale structures that are too much clustered. The anisotropies of the cosmic micro-wave background measured by COBE grandly favor Cold dark matter , since they give $\delta T/T \sim 10^{-6}$. But in a theory with only Cold dark matter it is difficult to correlate the observed velocity dispersion at small scales, of order $\sim few$ Mpc, with the COBE measurements at large scales, of order ~ 1 Gpc.

This rules out the theories with only Hot or Cold dark matter, but models with mixed dark matter, 30% of Hot dark matter and 60–70% of Cold dark matter and an other kind of candidate, or with a lot of Massive dark matter which decay into relativistic species, seem more promising.

Some models assume that after the inflation $\Omega = 1$ and that the dark matter is composed by an unstable WIMP which decay into relativistic particles leaving $\Omega = 1$. These unstable WIMPs could form the galaxies and the decay products clustered in bound structures only up to $\Omega = 0.2$. Such a model describe successfully the galaxies formation without disturbing the 2.73 K micro-wave background but require a fine tuning of the decay epoch which gives a too young age for the universe (Trimble 1990).

Miscellaneous

There exist various other dark matter candidates:

- A gravitational constant G increasing monotonically with the radius r or the acceleration $\ddot{r}$ can mimic the dark matter, making the luminous matter acting as bigger amount of dynamical mass. But some experiences seem indicate that G varies on distance much smaller than these required by the dark matter (Newman 1983).
- A non-vanishing cosmological constant Λ will act homogeneously on the universe and make the deceleration parameter q_0, and Ω_0 independent. So the measured value of Ω_0 may not be the exact value of Ω but a linear combination of Ω, Λ and $q_0 : \Lambda = 3H_0^2(\Omega/2 - q_0)$.

The Table 2 gives a sum-up of these candidates assuming $\Omega_{Baryons}/\Omega_{darkmatter} \sim 0.1$.

Now I study the super-symmetric partners as candidate for the dark matter.

5 – THE SUPERSYMMETRICAL THEORY

5.1 – Why a new particle physics theory ?

For energy less than 1 TeV two out of the four interactions, that exist in the Nature, are successfully described by the Weinberg-Salaam theory or the electro-weak theory. As indicated by is name this theory unifies the electromagnetism theory, *i.e.*, the interactions of charged particles, and the weak interactions, *i.e.*, the interactions responsible for the β decay. This theory described by the structure $SU(3)_C \otimes U(1)_{EM}$ (*cf.* Appendix I for more technical details) makes very successful predictions. But there are some hints that some *new* physics lies beyond this model.

The most convincing arguments are:

- Why are there three coupling constants: one for the strong interaction g_S, one for the weak process g' and one for the hyper-charge g ?
- The electro-weak theory does not treat the quarks and the leptons in the same way. So this theory cannot explain the equality between the absolute value of the charge of the proton and the electron $|q(p^+)| = |q(e^-)|$.
- Why are there only three families ? Some recent experiments at LEP tend to rule out a fourth family.
- There are too much parameters in this theory: the quark masses, the Weinberg's angle, ...
- Why is the theory left-handed ? Because right-handed neutrinos does not exist there is a (mathematical) disymmetry between left-handed fermions and right-handed fermions.
- Why is above 246 GeV the complete $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry restored ?

- There is no room for the gravitational forces in the electro-weak theories.
- *etc.*...

All these questions show that there **must** be something beyond the electro-weak theory.

5.2 – The Supersymmetry

The physics report from Haber and Kane (1985) is a good introduction to the supersymmetrical theory for physicists who have a background on particle physics. In this report the authors have put the stress on the signatures of the different particles and the different means which may probe this theory. But for peoples interested in the conceptual details of this theory, I referre to the physics report from Fayet and Ferrara (1977).

Various extensions of the electro-weak theory were proposed, like the Peccei-Quinn symmetry to solve the strong-CP problem of Quantum ChromoDynamics, Majorama models to give a mass to the neutrinos, Right-handed neutrinos, ... Among these the supersymmetry, which is an extension of the Poincaré group, seems preferred because this theory deals with the next range of energy 100 GeV–1 TeV and could be tested soon by the LEP, the Z^0 device SLC, the $\bar{p}p$ collider TEVATRON, the new European collider LHC, ... This theory allows a symmetry between fermions and bosons. Moreover the graviational interaction seems to appear "naturally" in a supersymmetrical theory depending of the position in the space-time. This theory is very beautiful and interesting, *i.e.*, it use a lot of mathematical tools and is physically not understood.

As I said the supersymmetry restores the symmetry between bosons and fermions, each known particle of spin s has a super-partner which spin differs by half an unit $s \pm 1/2$ and has the same other quantum numbers. So there is the same number of bosonic degrees of freedom than fermionic degrees of freedom. But since no super-partners are yet observed we must lift the degeneracy by breaking the electro-weak symmetry (*cf.* appendix I), and the super-partners must have a mass about 250 GeV greater than the particles masses. But, for technical reasons pointed out by Fayet (Fayet *et al.* 1977), to achieve this symmetry breaking we need two Higgs doublets H_1 and H_2. With one Higgs doublet only quarks of given charge can acquire mass.

The Table 3 gives a sum-up of all the super-partners in the case of a minimal model (1-dimensional theory with three majors parameters $\mathrm{tg}\,\beta$, M_2 and μ).

In supersymmetry there is some mixing between the different super-partners, thus the mass eigenstates, *i.e.*, the eigenstates with give term like $m\bar{\phi}\phi$ in the lagrangian are not the interaction eigenstates, *i.e.*, the particles detected during experimentations in a detector. Loosely speaking, a classical way to see the difference between the eigenstates of masses and the eigenstates of interactions is to study the case of an hypothetical system of mass $\mathcal{M}$ made by a neutral particle $\mathcal{P}_1$ and a positively charged particle $\mathcal{P}_2$ (such a system could be a neutron and a proton bounded by strong forces). I assumed that in absence of electromagnetic forces the sytem $\mathcal{P}_1 \cup \mathcal{P}_2$ is strongly bounded, and that the link between the two particles is weaker than the electromagnetic interaction. We can see this liaison as a spring with a natural length ℓ_0 very small and a repelling constant k so strong that when there is no electromagnetic forces $\mathcal{P}_1$ and $\mathcal{P}_2$ are stuck together, and k weak enough so that the electromagnetic force is greater than $k\ell_0$. In absence of any forces our two particles could be identified, because $\ell_0 \ll 1$, we only detect *one* particle, it is the so-called eigenstate of mass. But when there is some electromagnetic forces, the distance between the two particles is no longer ℓ_0. And we can can distinguish the *two* particles $\mathcal{P}_1$ and $\mathcal{P}_2$, they are what we called the eigenstates of (electromagnetic) interaction .

This occurs in supersymmetry with the charged bosons $\widetilde{H}^{\pm}_{1,2}$, $\widetilde{W}^{\pm}$ which mix in charginos, and with the neutral bosons $\widetilde{W}^3$, $\widetilde{B}$, $\widetilde{H}^0_{1,2}$ which mix in neutralinos. Since I suppose that the neutralino, noted $\widetilde{\chi}$, is a dark matter candidate, hereafter I only speak about this super-particle.

After the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ breaking, appear some mixing parameters between the super-partners of the gauge bosons of the electro-weak interaction and the Higgs W^3, B^0, H^0_1 and H^0_2. [7] The new mixing parameters (I deal only with the minimal set of parameters) M_1 the mass of the Bino $\widetilde{B}$, M_2 the mass of the Wino $\widetilde{W}^3$, μ the mixing parameter of the Higgsinos $\widetilde{H}^0_{1,2}$, arise necessarily from the spontaneous symmetry breaking and are not constrained. Grand unification theories (the theories which try to unify *all* the interactions: the electromagnetic interactions, the weak interactions, the strong interactions and the gravitation) give the relation $M_1 = 5/3\, M_2 \,\mathrm{tg}^2\,\theta_w$ between M_1 and M_2. [8] There is one other unconstrained parameter $\mathrm{tg}\,\beta = v_2/v_1$ the ratio of the vacuum expectation value of the two Higgs $v_1 = \langle H_1 \rangle$ and $v_2 = \langle H_2 \rangle$ (in fact only technical reasons give an upper bound fixed by the ratio of the mass of the Top-quark and the bottom quark $1 < \mathrm{tg}\,\beta < m_t/m_b$) (DERUJULA *et al.* 1992). The W^3, B^0, H^0_1 and H^0_2 are mixed by the matrix (which I give without more explanation)

$$\left(\widetilde{W}^3 \ \widetilde{B}^0 \ \widetilde{H}^0_1 \ \widetilde{H}^0_2 \right) \begin{pmatrix} M_2 & 0 & m_Z \cos\theta_w \cos\beta & -m_Z \cos\theta_w \sin\beta \\ 0 & M_1 & -m_Z \sin\theta_w \cos\beta & m_Z \sin\theta_w \sin\beta \\ m_Z \cos\theta_w \cos\beta & -m_Z \sin\theta_w \cos\beta & 0 & -\mu \\ -m_Z \cos\theta_w \sin\beta & m_Z \sin\theta_w \sin\beta & -\mu & 0 \end{pmatrix} \begin{pmatrix} \widetilde{W}^3 \\ \widetilde{B}^0 \\ \widetilde{H}^0_1 \\ \widetilde{H}^0_2 \end{pmatrix}$$

The eigenstates are $\widetilde{\chi} = Z_{i1}\,\widetilde{W}^3 + Z_{i2}\,\widetilde{B}^0 + Z_{i3}\,\widetilde{H}^0_1 + Z_{i4}\,\widetilde{H}^0_2$ for $i = 1, 4$.

We can see that if $M_1 = M_2 = 0$, 0 is an eigenvalue with the associated eigenstate $\sin\theta_w\ \widetilde{W}^3 + \cos\theta_w\ \widetilde{B}^0$ which is the photino $\widetilde{\gamma}$, the super-partner of the photon. And if one takes μ equal to 0, the lightest state, again massless, is a higgsino $Z_{i3}\,\widetilde{H}^0_1 + Z_{i4}\,\widetilde{H}^0_2$.

On the figures 1 and 2, I show a projection of the mass of the LSP in the M_2 *vs.* μ for

[7] B^0 is a SU(2) singlet, $H_1 = \left(H^0_1, H^-_1\right)$ and $H_2 = \left(H^+_2, H^0_2\right)$ are the two Higgs doublets that give two scalar fields h and H, a pseudo-scalar, and two charged Higgs bosons $H^{\pm}$ *cf.* Appendix I.

[8] θ_w is the Weinberg angle, it arises when the symmetry $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$ is broken to $\mathrm{U}(1)_{EM}$, this angle relates the value of the coupling constants g of $\mathrm{SU}(2)_\mathrm{L}$ and g' of $\mathrm{U}(1)_\mathrm{Y}$ by the relation $\mathrm{tg}\,\theta_w = g'/g$ the experimental value given by $p\bar{p}$ experiments at PEP or PETRA gives $\sin^2\theta_w = 0.2315 \pm 0.0003$ for a Top-quark mass $m_t = 100$ GeV, value taken from the PARTICLE PROPERTIES DATA BOOKLET JUNE 1992.

$50 \leq M_2 \leq 10^4$ GeV and $50 \leq \mu \leq 10^4$ GeV. I have superposed the fraction of gaugino $Z_{n1}^2 + Z_{n2}^2$. The figure 1 is plotted for $\mu > 0$ and the figure 2 for $\mu < 0$.

Thus very light super-partners are possible. Among all the super-particles the lightest is the most interesting since it was showed that this ordinary particle is stable because a multiplicative number, the R-parity, is conserved until the baryon and lepton numbers are exactly conserved. The "ordinary" particles have R = 1 and the super-particle R = −1. So an initial state composed only by "ordinary" particles can give a final state containing only an even number of super-particles, and the decay products of a super-particle must contain an odd number of super-particles. Thus the lightest supersymmetrical particle, the LSP, is stable since it cannot decay in an other super-particle, because it is too light, and in "ordinary" particle, because of the R-parity.

So the LSP seems a good candidate for the dark matter. In the following analysis I will study the case where the LSP is the neutralino and is a candidate for the dark matter.

6 – DETECTION OF THE LSP

If the LSP is a component of the dark matter a non-negligible fraction must have survived the Big Bang. This fraction is a vestige of the earliest times of the universe. So I first explain how to calculate this relic density (a more detailed analysis is given in the Appendix II). Then I review two different kinds of detection. But I only detail the indirect detection which was the purpose of my work at Berkeley.

6.1 – The relic density

For the problem of the relic density of any particle, there are two main books, one from Kolb and Turner (1990) and the monography from Bernstein (1988).

In the earliest times of the universe the matter and the radiation were in equilibrium. There were constant exchanges between them. The matter annihilated but the temperature was high enough to "create" some matter. For example, consider the electron e^- and its anti-particle the positron e^+, I assume that there is no disymmetries between a particle and its anti-particle. The electrons and positrons annihilated by, say $e^+e^- \rightarrow 2\gamma$, but since the temperature was high enough e^+e^- pairs were created. And there were an equilibrium between the electron-positron system and the photons.

But as the universe expand, it is cooling and some species are going out of equilibrium. We can have an order of the time when the decoupling occurred, for a given species, by comparing its free mean path to the size of the universe, which is about H^{-1}. If its free mean path is greater than H^{-1} the particle is still coupled with the radiation. But when its free mean path becomes lower than H^{-1} the particle gets out of equilibrium. [9]

Another way to quantify the epoch of decoupling is to compare the temperature of the universe with the rest mass of the particle, since this is the minimum of energy needed to create this particle. So if the temperature is greater than the rest mass of the particle this species is in equilibrium and when the temperature drops below its rest mass the particle gets out of equilibrium. The temperature of decoupling T_f depends of the cross-section of the annihilation process of the particle, but we can consider that $T_f \simeq m/20$ for the LSP.

For example, the decoupling temperature for the electron is about $T_f \sim 10^9$ K, which correspond to $t \sim 4$ sec after the Bang. Since this time the electrons in the universe annihilate into photons γ.

The constituent of the dark matter (which interested us: the LSP) is a relic of this early epoch and its actual density is an image of the state of the universe when the LSP decoupled (this is the reason why many physicists study the cosmic micro-wave background, because it is an image of the universe 200,000 years after the Big Bang, see section 3). The problem is to calculate how many neutralinos $\widetilde{\chi}$, our dark matter candidate, survived this epoch. This problem can be solved in the frame of the statistical mechanics (*cf.* Appendix II). Here I will just give the only results needed for a general understanding.

The number density of LSP $n_{\widetilde{\chi}}$ verifies the following equation:

$$\frac{dn_{\widetilde{\chi}}}{dt} + 3Hn_{\widetilde{\chi}} = -\langle\sigma|v|\rangle \left[n_{\widetilde{\chi}}^2 - (n_{\widetilde{\chi}}^{eq})^2\right].$$

The second term of the left-hand-side of this equation is the dilution due to the expansion of the universe. And the right-hand-side of this equation is the term due to the annihilation of the particle $\widetilde{\chi}\widetilde{\chi} \to f\bar{f} + \cdots$, $\langle\sigma|v|\rangle$ is the average cross-section (see appendix III for a definition). And $n_{\widetilde{\chi}}^{eq}$ is the number density of $\widetilde{\chi}$ at equilibrium. The remaining density of LSP, that is how much there are today, $n_{\widetilde{\chi}}^{\infty}$ is generally expressed in term of the ratio of the density and the entropy by co-volume (see appendix III for a definition of the co-volume) $Y^{\infty} \widehat{=} n_{\widetilde{\chi}}^{\infty}/s$. The entropy by co-volume is related to the temperature of the universe by $s = 2\pi^2 g_* T^3/45$ (g_* is the number of effectively relativistic degrees of freedom). According to Kolb and Turner the present value of $s_0 = 2970\,\mathrm{cm}^{-3}$ if we assume that $T = 2.75\,\mathrm{K}$. The relic density is given by

$$\Omega_{\widetilde{\chi}} h^2 \widehat{=} \frac{\rho_{\widetilde{\chi}}^{\infty}}{\rho_c/h^2} = \frac{m_{\widetilde{\chi}} n_{\widetilde{\chi}}^{\infty}}{\rho_c/h^2} = \frac{Y^{\infty} s_0 m_{\widetilde{\chi}}}{\rho_c/h^2}.$$

Or

$$\Omega_{\widetilde{\chi}} h^2 \simeq 2.82 \times 10^8\, Y^{\infty}\ (m_{\widetilde{\chi}}/\mathrm{GeV})$$

[9] Because the scale factor of the universe R increases and its derivative with respect of the time $\dot{R}$ decreases thus H decreases (In fact this is always the case for an open and flat universe, and for a close universe this true only before the beginning of the recontraction phase but this phase starts 40×10^9 years after the Bang).

The figure 3 represents the relic density $\Omega_{\widetilde{\chi}} h^2$ versus the mass of the LSP. This graph was drawn with $\mathrm{tg}\,\beta = 2$, the masses of the Top-quark $m_{top} = 130\,\mathrm{GeV}$ and the mass of the lightest higgs $m_{h_2^0} = 50\,\mathrm{GeV}$. Because I took the infinite squark masses I suppressed some channels and this is the reason of the big values, $1 \ll \Omega_{\widetilde{\chi}}$, for the relic density when the mass of the LSP is low and is a gauginos. There are two way of considering such a graph. We can assume that the mass of the LSP is fixed to a given value (say $m_{\widetilde{\chi}} \sim 100\,\mathrm{GeV}$) and deduce the value of Ω_0. Or to use the constraints on the values of Ω_0, $0.2 \lesssim \Omega_0 \lesssim 1$ and calculate the masses allowed for the LSP. Recall that the lower limit on Ω_0 comes from the observations of the universe, see section 2 and Table 1, and the upper limit for Ω_0 comes from the wish that the universe is not over-closed, in fact not close at all since it seems that the universe is unclosed and unbound (for example, see the measured values of q_0 Weinberg 1972). In fact theses two points of view can be taken, since the two models the supersymmetrical theory and the Friedmann-Robertson-Walker's cosmology have unknown parameters. So as there is a connection between these two theories, one can use one of them to determine the parameters of the other. The indirect detection group, namely Bernard Sadoulet, they constrain Ω_0 between 0.2 and 1 and try to deduce the values of the unknown parameters of the supersymmetrical theory ($\mathrm{tg}\,\beta$ and the different masses of the super-symmetrical particles).

As I said in section 5.2 the LSP is stable so how can we detect it ? There are two methods: the *direct detection* and the *indirect detection*.

6.2 – Direct detection

For an analysis of the experimental problems of the direct and indirect detections, see the article from Primack *et al.* (1988).

The direct detection group tries to detect the LSP with detectors in a laboratory. This method is based on the interactions between a LSP and the nuclei of the detector. When the LSP get through the detector it interacts with the quarks by $\widetilde{\chi} + q \rightarrow \widetilde{\chi} + q$. The recoil of the nucleus, typically ^{68}Ge or Si, ionises the cristal of the detector by small currents, induces a small increase of temperature and a phonon shower. This effect is very small, the typical energy of these interactions is a few TeV, so the detectors must be keep at low temperatures, and the experimentalists try to avoid the background noise, *e.g.*, ambient radioactivity, cosmic rays, …But the neutralino has a peculiar signature since its interactions with nuclei produce lower ionisations. Moreover the motion of the Earth on its orbit will produce a detectable annual modulation.

6.3 – Indirect detection

This method is to detect the energetic neutrinos, their energy is about $E_\nu \simeq 0.5 m_{\widetilde{\chi}} \gtrsim 1$ GeV, produced by this annihilation of the LSP by $\widetilde{\chi}\widetilde{\chi} \to \delta + \cdots$. These annihilations occurs when the LSPs are captured in an astrophysical body, like the Sun or the Earth and enough concentrated. When the LSP is captured, it is more concentrated and can annihilate. This annihilation gives some fermions-induced energetic neutrinos, which the indirect detection group tries to detect. For example at Irvin-Michigan-Brook-haven (IMB), Fréjus or Kamiokande, they try to detect the up-ward muons produced by the interactions of the neutrinos with the rock below the detector. These detectors have only a size of $400\,\mathrm{m}^2$ but newer detectors are $1000\,\mathrm{m}^2$ wide for MACRO and $10^6\,\mathrm{m}^2$ wide for the detector Antartic Muons And Neutrinos Dectector Array, which is under the ice of the south pole, and DUMAND, which is in the Pacific ocean. The figure 4 gives the estimated size of the detectors for 4 events per year.

As explained in section 6.5 experimentalists try to detect the muon induced by the interaction of the neutrino with the rock below the detector.

6.4 – Annihilation rate in the Sun and in the Earth

The problem of the capture of the LSPs by an astrophysical body was run out by Gould in his papers (Gould 1987).

As I said in section 2.1 the dark matter is present at every scale, so there is some dark matter in the Solar system. All the detection of dark matter technics try to detect the dark matter Halo around the Earth. The LSPs in the neighborhood of our planet interact with the Earth or the Sun. If after interaction the speed of the LSP is lower than the escape velocity of the astrophysical object, the particle is captured.
The number of LSP captured $n_{\widetilde{\chi}}$ verifies the equation

$$\dot{n}_{\widetilde{\chi}} = C - C_A n_{\widetilde{\chi}}^2 \tag{2}$$

where C is the capture rate of the LSPs by the Sun or the Earth, and C_A is the annihilation for the particle at *zero* relative velocity (recall that the neutralino is a Cold dark matter candidate) $C_A \propto \langle\sigma|v|\rangle$. The annihilation rate by unit of time for *one* particle, is $\Gamma_A = (C_A n_{\chi}^2)/2$.
By solving the equation (2)one finds that

$$\Gamma_A = \frac{C}{2}\,\mathrm{th}\left(\frac{t}{\tau_A}\right) \tag{3}$$

with the time scale factor for the capture rate and annihilation rate $\tau_A = (CC_A)^{-1/2}$.
As the age of the Sun is about $t_\odot \sim 1.5\times10^{17}$ sec and that for the sun $\tau_A \sim 10^{16}$ s the annihilation rate is maximum $\Gamma_A = C/2$. But for the Earth $t_\oplus \sim 1.4\times10^{17}$ s and $\tau_A \sim 10^{24}$ s (because the capture rate is lower than for the Sun, see the discussion below) so the annihilation rate is lower

than the capture rate $\Gamma_A \simeq 10^{-9} C$. Moreover τ_A for the Earth is very much lower than for the Sun, because there is less LSP captured in the Earth.

We have to determine the capture rate of the LSPs by the Earth or the Sun. This is not very difficult since the distribution of the LSPs in the Halo is independent of the time. We just have to calculate the number of LSPs which speed is lower than the liberation speed after scattering on the nucleus of the constituent of the Sun or the Earth. The distributions of these LSPs in the Sun or the Earth is given by a Maxwell's distribution. For the LSPs captured in the core of the Earth, it is assumed that they are described by a Maxwell's distribution with a temperature in the range $4.5\text{–}5.5 \times 10^3$ K. The exact abundances of the elements in the Earth is not well known. There are some discrepancies about the fraction of total Earth mass which is due to core elements, the model from Stacy 1977 has an abundance of Fe of 24%, Ni of 3% and S of 5%, the model from Ringwood 1979, which I chose in the program, has Fe 26%, Ni 3% and S 3%.

The velocity dispersion of the LSPs in the Halo is $v_{\tilde{\chi}} \sim 300\,\text{km/s}$, the liberation speed of the Sun $v_{sun} \sim 618\,\text{km/s}$ and for the earth $v_{earth} \sim 11\,\text{km/s}$. These figures show that the capture in the Sun is very effective and the capture in the Earth is more difficult. But in the Earth, when the LSP scatters on a nucleus which mass matches the LSP mass, it loose a lot of its energy and is captured. Before giving the right formula for the capture rate, I discuss the how we can quantify the rate of LSPs captured in an astrophysical body.

I recall that the in the Halo of our galaxy the LSPs have a velocity $v_{\tilde{\chi}}$. Thus when a neutralino passes next to a body, it will be captured if its speed is lower than the escape velocity of the body. But particles with velocities greater than the escape velocity can be captured if they pass through the astrophysical body. If a particle passes through the body and scatters on a nucleus with a mass m_i close to the mass $m_{\tilde{\chi}}$ of the particle, the latter loses a lot of kinetic energy in the collision. Hencefore the velocity of the particle after the scatter can be less than the escape velocity of the body, and the particle captured. In fact, the exact parameter which quantifies the capture rate is

$$A = \frac{3}{2} \frac{m_{\tilde{\chi}} m_i}{(m_{\tilde{\chi}} - m_i)^2} \left(\frac{v_{esc}}{\bar{v}}\right)^2 \phi_i \tag{4}$$

where $m_{\tilde{\chi}}$ is the mass of the LSP, m_i the mass of the nucleus, v_{esc} the escape velocity, $\bar{v}$ the velocity dispersion of the LSPs in the Halo, $\bar{v} \mathrel{\widehat{=}} 3T/m_{\tilde{\chi}}$. Following Gould (1987) I took $\bar{v} = 300\,\text{km}\,\text{s}^{-1}$ and $\phi_i \mathrel{\widehat{=}} v^2/v_{esc}^2$ the gravitational potential divided by the value at the surface of the body. For the Sun $\langle\phi_i\rangle = 3.3$ and for the Earth $\langle\phi_i\rangle = 1.2$ and 1.6. [10]

As expectected after the previous analysis of the capture, this quantity depends on the ratio of v_{esc} and $\bar{v}$, and the inverse of the difference of the masses of the neutralino $m_{\tilde{\chi}}$ and the mass of the target nucleus m_i. Thus when $1 \ll A$ ($m_{\tilde{\chi}} \simeq m_i$ or $\bar{v} \ll v_{esc}$), there is no suppression and

[10] For the Earth the problem is different since our planet is divided in a core and a mantle. In the core $\langle\phi_i\rangle = 1.6$ and in the mantle $\langle\phi_i\rangle = 1.2$.

the capture is effective. And when $A \ll 1$ ($v_{esc} \ll \bar{v}$ and $m_{\widetilde{\chi}}$ does not match any masses m_i) the capture rate is suppressed by a factor A.

A complete calculation, done by Gould in (Gould 1987), gives the exact expression of the capture rate

$$C = \sum_{specieses} \left[\left(\frac{8}{3\pi} \right)^{\frac{1}{2}} \sigma \bar{v} n_{\widetilde{\chi}} \right] \left[\frac{M_b}{m} \right] \left[\frac{3 v_{esc}^2}{2\bar{v}^2} \langle \phi \rangle \right] [\xi_\eta(\infty)] \left[\left\langle \frac{\phi}{\langle \phi \rangle} \left(1 - \frac{1 - e^{-A^2}}{A^2} \right) \frac{\xi_\eta(A)}{\xi_\eta(\infty)} \right\rangle \right]$$

The sum runs over all the kind of nuclei in the body.

The first term is the interaction, described by the cross-section σ, of the LSP with a nucleus of mass m. The second term is the fraction of nucleus in the body which mass is M_b. The third term is a focusing factor which gives how many particles are captured by the gravitational potential of the body: $\langle \phi \rangle$ is the average value over all the species in the body of the gravitational relative potential ϕ. This term is an escape velocity averaged over all the species and the volume of the body divided by the thermal velocity dispersion $\langle v^2 \rangle / \bar{v}^2$. It gives how many LSPs are captured (if $\bar{v}$ is greater than $\langle v^2 \rangle$ the capture is suppressed). The fourth term due to the motion of the body is the ratio of the differential capture rate for an observer at rest with respect to the thermal distribution and the capture of an observator with a dimensionless velocity η. The fifth term is the suppression factor I discussed before.

In this explanation the Earth was assumed isolated, I did not considerated the influence of the gravitational potential of the Sun. The effects of this potential are believed to be important but very difficult to calculate. A LSP which velocity is greater than the escape velocity of the Earth can interact with it, loose some energy and escape from the Earth, but under the influence of the gravitational potential of the Sun be recaptured by our planet.

There is another effect that I want to discuss, it is the influence of the form factor of the nuclei. When the LSPs have large velocity they see the interior of the nucleus with which they interact (more precisely, this occurs if the impulsion, q, of the LSP is greater than $\hbar/R$, where R is the root mean square of the nucleus radius). The form factor of the nucleus which is the Fourrier transform of the distribution of masses in the nucleus $F(q) = \int \rho(x) e^{i\mathbf{q}\cdot\mathbf{x}}\, d\mathbf{x}$, is given by (Gould 1987) $|F(q^2)|^2 = e^{-q^2 R^2 / 3\hbar^2}$. This effect is believed to be important for the heavy LSPs captured in the Sun, but negligible ($|F(q^2)|^2 \ll 1$) if they are captured in the Earth.

In the formula for the capture rate appears the cross-section of the interaction of the LSP with one nucleus. Basically, there are two kinds of interactions, the *spin dependent* interactions and the *scalar* or spin independent interactions. The spin dependent interactions depend, as indicated by their name, on the couplage between the spin $1/2$ of the neutralino, and the spins of the nucleus. In the Earth there are not enough nuclei with spin so this kind of interaction is negligible, but in the Sun this kind of interaction occurs with the very abundant Hydrogen. Thus if the neutralino has only spin dependent interaction there will be no signal from the Earth.

6.5 – Neutrino spectra from LSP annihilation

The spectra of the neutrinos produced by annihilation of fermions were calculated by Ritz and Seckel (1991), and references therein. The article writen by Kamionkowski gives a sum-up of all the relevant results (Kamionkowski 1991).

Given the annihilation rate Γ_A (*cf.* previous section) the differential flux of neutrino ν of type i, *e.g.*, ν_e,ν_μ,..., is

$$\left(\frac{d\phi}{dE}\right)_i = \frac{\Gamma_A}{4\pi R^2} \sum_f B_f \left|\frac{dN}{dE}\right|_{fi}$$

where R is the distance from the source and B_f the branching ratios for the process $\widetilde{\chi}\widetilde{\chi} \to f\bar{f}$ where f is the parent of the neutrino , and $dN/dE|_{fi}$ is the energy distribution of the type i-neutrino produced by the fermions f. The sum runs over the different channels by which the LSP can decay.

There are two methods for detecting the signal from the annihilation of the neutralinos. First the direct detection of the neutrinos by contained events in the detectors. The neutrinos are expected to be detected by their interactions with of the medium of the detector. The cross-section of these contained events is proportional to the energy of the neutrino (Kamionkowski 1991). Second, we can detect the up-ward neutrino-induced muons. Actually the light muons and light quarks are stopped in the core of the Earth. But the neutrinos interact with the nuclei in the rock and produce some muons in the rock below the detector. The range of these neutrino-induced muons is proportional to the energy of the muon, as the cross-section of the neutrinos with the nuclei is proportional to the energy of the neutrino then the probability to detect an induced muon is proportional to the energy squared of the neutrinos (Kamionkowski 1991). Thus the signal from neutrino-induced event is stronger than the signal from contained events for high neutrinos energy. Moreover to calculate the detect rate we only need the second-moment of the neutrino distribution

$$\langle Nz^2 \rangle = \frac{1}{m_{\widetilde{\chi}}} \int \frac{dN}{dE}\bigg|_{fi} E^2 \, dE$$

where $dN/dE|_{fi}$ is the energy distribution of the type i-neutrino produced by the fermions f. The detect rate for neutrino-induced throughgoing muons event is (Kamionkowski 1991)

$$\Gamma_{detect} = 1.27 \times 10^{-29} \, C \, m_{\widetilde{\chi}}^2 \sum_i a_i \, b_i \, \sum_f B_f \, \langle Nz^2 \rangle \, \mathrm{m}^{-2} \, \mathrm{yr}^{-1}$$

for the neutrinos from the Sun. For the Earth the expression is the same but multiplied by the square of the ratio of the distance between the Earth and the Sun to the Earth radius 5.6×10^8. C is the capture rate in unit of s^{-1}. The sum on i runs over the muon neutrinos and anti-neutrinos and the sum on f is over all the fermionic parents, and B_f are the branching ratios for the process which give the fermion. For example, the annihilation of two neutralinos $\widetilde{\chi}$ into ZH_i^0 final states which annihilate into some fermions with the branching ratios B_f. And these fermions decay into some neutrinos or anti-neutrinos and anything else.

This detection rate is proportional to the capture rate C. So as I said in section 6.5 if the neutralino has only spin-dependent interactions there will be absolutly no signal from the Earth since there is not enough nuclei with spin in it. But when the energy of the LSP is in the range $10\,\mathrm{GeV} \lesssim m_{\widetilde{\chi}} \lesssim 75\,\mathrm{GeV}$, which is the range of the masses of the nuclei in the Earth, the signal from our planet is belived to be greater than which from the Sun (this comes from the equation (4)with $m_{\widetilde{\chi}} \simeq m_i$).

If the capture rate and the annihilation rate for the Earth are stronger, τ_A decreases and the signal from the Earth is higher (see equation (2)). Moreover the interaction of the neutrinos with the medium of the Sun deplete the signal (Kamionkowski 1991). Hencefore s such case the signal from the Earth could dominate. But in the contrary the signal from the Earth is weaker. It is expected that 98% of the signal from the Earth will be detected in an angle of 14° for 20 GeV neutralinos (Gould 1987).

7 – FINAL COMMENTS

Since the experimentations have not yet started, I cannot give any estimates of the real density of neutralinos in the universe. But some results should come very quickly from the direct and indirect detection experiments and from the new colliders, *e.g.*, the LHC, which will confirm or definitively rule out the supersymmetrical theory. But I can give some orders of the capture rate in the case of an energy-density of LSP in the halo of $\rho = 0.4\,\mathrm{GeV/cm}^3$, *cf.* figures 3, 5 and 6. If we want to observe four events a year the lowest size of the detector is $10^5\,\mathrm{m}^2$ *cf.* figure 7.

8 – ACKNOWLEDGMENTS

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9 – TABLES AND FIGURES

This Chapter contains all the figures and the tables cited in the text.

9.1 – Tables

Table 1 Cosmological densities estimates (Primack *et al.* 1988).

	Scale ($\times h^{-1}$ Mpc)	Ω_0
Luminous parts of galaxies	~ 0.02	0.01
Halos of galaxies and groups of galaxies	~ 0.1–1	0.02–0.2
Cosmic virial theorem	~ 3	~ 0.2
Virgo infall	~ 10	~ 0.2
Large-scale infall	~ 30	~ 0.2–1
Cosmological tests	3000	0.1–2
Cosmic inflation		1

This table gives a summary of the estimated values of Ω_0 for different scales varying between the size of the galaxies and the large-scales $\sim 30\,h^{-1}$ Mpc. As we can see the mean value of Ω_0 is around 0.2. The last line of this table give the value expected by theoricians from the different cosmological models. As said in the text the value $\Omega_0 = 1$ is grandly favored.

Table 2 Summary of nonbaryonic dark matter candidates[a] taken from Trimble (1990).

Candidate/particle	Approximate mass	Predicted by	Astrophysical effects
G(r)	—	Non-Newtonian gravitation	Mimics DM on large scales
Λ (cosmological constant)	—	General relativity	Provides $\Omega = 1$ without DM
Axion, majoron, goldstone boson	10^{-5} eV	QCD ;PQ symmetry breaking	Cold DM
Ordinary neutrino	10–100 eV	GUTs	Hot DM
Light higgsino, photino, gravitino, axino, sneutrino[b]	10–100 eV	SUSY/SUGRA	Hot DM
Para-photon	20–400 eV	Modified QED	Hot/warm DM
Right-handed neutrino	500 eV	Superweak interaction	Warm DM
Gravitino, etc.[b]	500 eV	SUSY/SUGRA	Warm DM
photino, gravitino, axino, sneutrino, mirror particle, simpson neutrino[b]	keV	SUSY/SUGRA	Warm/cold DM
Photino, sneutrino, higgsino, gluino, heavy neutrino[b]	MeV	SUSY/SUGRA	Cold DM
Shadow matter	MeV	SUSY/SUGRA	Hot/cold (like baryons)
Preon	20–200 TeV	Composite Models	Cold DM
Monopoles	10^{16} GeV	GUTs	Cold DM
Pyrgon, maximon, perry pole, newtorites, Schwarzchild	10^{19} GeV	Higher-dimension theories	Cold DM
Supersymmetric strings	10^{19} GeV	SUSY/SUGRA	Cold DM
Quark nuggets, nuclearites	10^{15} g	QCD, GUTs	Cold DM
Primordial black holes	10^{15-30} g	General relativity	Cold DM
Cosmic strings, domain walls	10^{8-10} $M_\odot$	GUTs	Promote galaxy formation, but cannot contribute much to Ω

[a] *Abbreviations: DM, dark matter ; QCD, quantum chromodynamics, PQ Pecci & Quinn ; GUTs grand unified theories ; SUSY supersymmetric theories ; SUGRA, supergravity ; QED, quantum electrodynamics.*

[b] *Of these various supersymmetric particles predicted by assorted versions of supersymmetric theories and supergravity, only one, the lightest, can be stable and contribute to* Ω*, but the theories do not at present tell us which one it will be or the mass to be expected.*

Table 3 Sum-up of the Supersymmetric particles and their partners (HABER *et al.* 1985).

Normal particles	Weak interaction eigenstates			Mass eigenstates			Mass eigenstates with specific coupling	
	Spin	Symbol	Name	Spin	Symbol	Name	Symbol	Name
$q = u, d, s, c, b, t$	$1/2$	$\widetilde{q}_{\mathrm{L}}, \widetilde{q}_{\mathrm{R}}$	scalar-quark	0	$\widetilde{q}_1, \widetilde{q}_2$	scalar-quark		
$\ell = e, \mu, \tau$	$1/2$	$\widetilde{\ell}_{\mathrm{L}}, \widetilde{\ell}_{\mathrm{R}}$	scalar-lepton	0	$\widetilde{\ell}_1, \widetilde{\ell}_2$	scalar-lepton		
$\nu = \nu_e, \nu_\mu, \nu_\tau$	$1/2$	$\widetilde{\nu}$	scalar-neutrino	0	$\widetilde{\nu}$	scalar-neutrino		
g gluons	1	$\widetilde{g}$	gluino	$1/2$	$\widetilde{g}$	gluino		
$\mathrm{W}^\pm$	1	$\widetilde{W}^\pm$	wino	$1/2$			$\widetilde{w}^\pm$	wino
H_1^+	0	$\widetilde{H}_1^+$	higgsino	$1/2$	$\widetilde{\chi}_{1,2}^\pm$	charginos	$\widetilde{h}^\pm$	higgsino
H_2^-	0	$\widetilde{H}_2^-$	higgsino	$1/2$			$\widetilde{\omega}_1, \widetilde{\omega}_2$	wiggsino
γ	1	$\widetilde{\gamma}$	photino	$1/2$			$\widetilde{\gamma}$	photino
Z^0	1	$\widetilde{Z}^0$	zino	$1/2$			$\widetilde{z}$	zino
H_1^0	0	$\widetilde{H}_1^0$	higgsino	$1/2$	$\widetilde{\chi}_i^0$	neutralinos	$\widetilde{h}_1, \widetilde{h}_2$	higgsino
H_2^0	0	$\widetilde{H}_2^0$	higgsino	$1/2$			$\widetilde{\zeta}_1, \widetilde{\zeta}_2$	ziggsino
$\begin{pmatrix} \mathrm{W}^3 \\ \mathrm{B} \end{pmatrix}$		$\begin{pmatrix} \widetilde{W}^3 \\ \widetilde{B} \end{pmatrix}$	(wino / bino)				$\begin{pmatrix} \widetilde{w}^3 \\ \widetilde{b} \end{pmatrix}$	(wino / bino)

i) The super-partner of a fermion is a boson which spin is 0 so they are scalar. We name them by adding the prefix "scalar" to the mass of the fermion, e.g., the super-partner of an electron is a scalar-electron or shorter a slepton.

ii) The super-partner of a boson is a fermion, its name is the name of the boson with the suffix "ino". For instance, the super-partner of the W is a wino and of the Higgs is a higgsino.

iii) Because of mixing, the eigenstates of mass (the eigenstates which give term like $m\bar{\psi}\psi$ in the lagrangian see section 5.2) are different of the eigenstates of interactions (what we detect). The examples are the charginos and the neutralinos. The charginos are a linear combination of the super-partners of the charged bosons $\widetilde{H}_{1,2}^\pm$, $\widetilde{W}^\pm$. The neutralino is a linear combination of the neutral super-partners of the bosons $\widetilde{W}^3$, $\widetilde{B}$, $\widetilde{H}_{1,2}^0$ (or of the $\widetilde{\gamma}$,$\widetilde{Z}^0$).

iv) The ziggsino is a sum of 50% of zino and 50% of higgsino and the wiggsino is 50% of gaugino (wino or bino) and 50% of higgsino.

9.2 – Figures

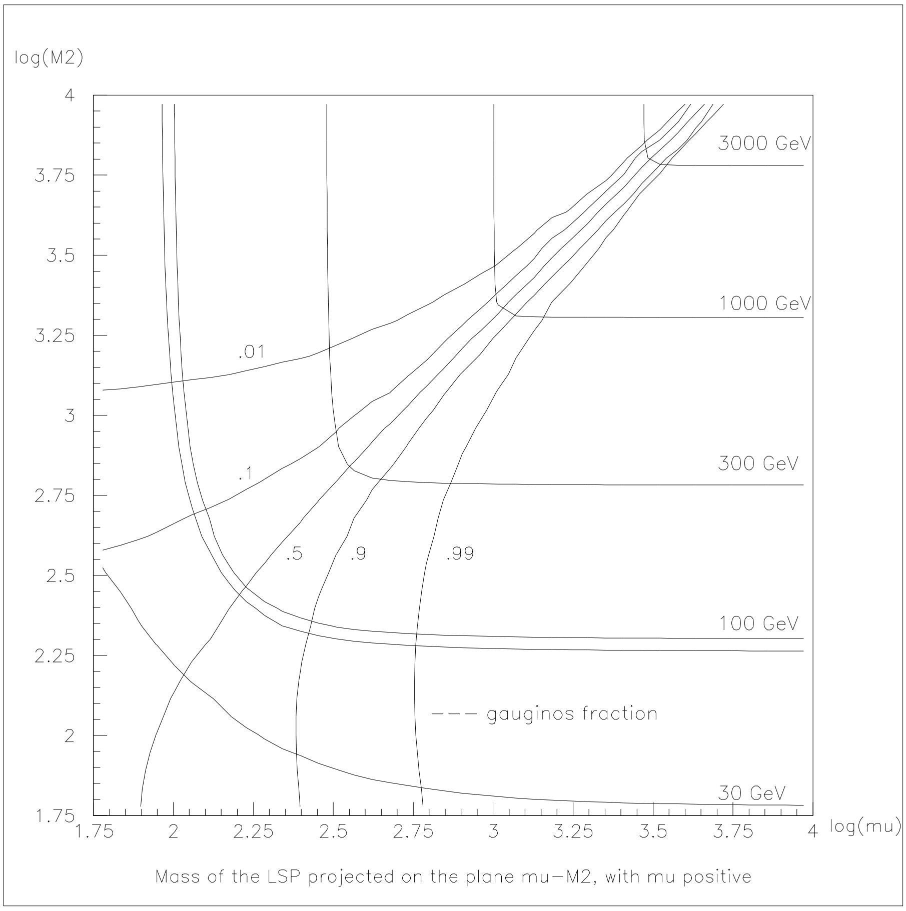

Figure 1 *This figure represents a projection of the mass of the LSP in the plan M_2 vs. μ for M_2 and μ varying in the range 50–10^4 GeV. I have superposed the fraction of gaugino $Z_{n1}^2 + Z_{n2}^2$*

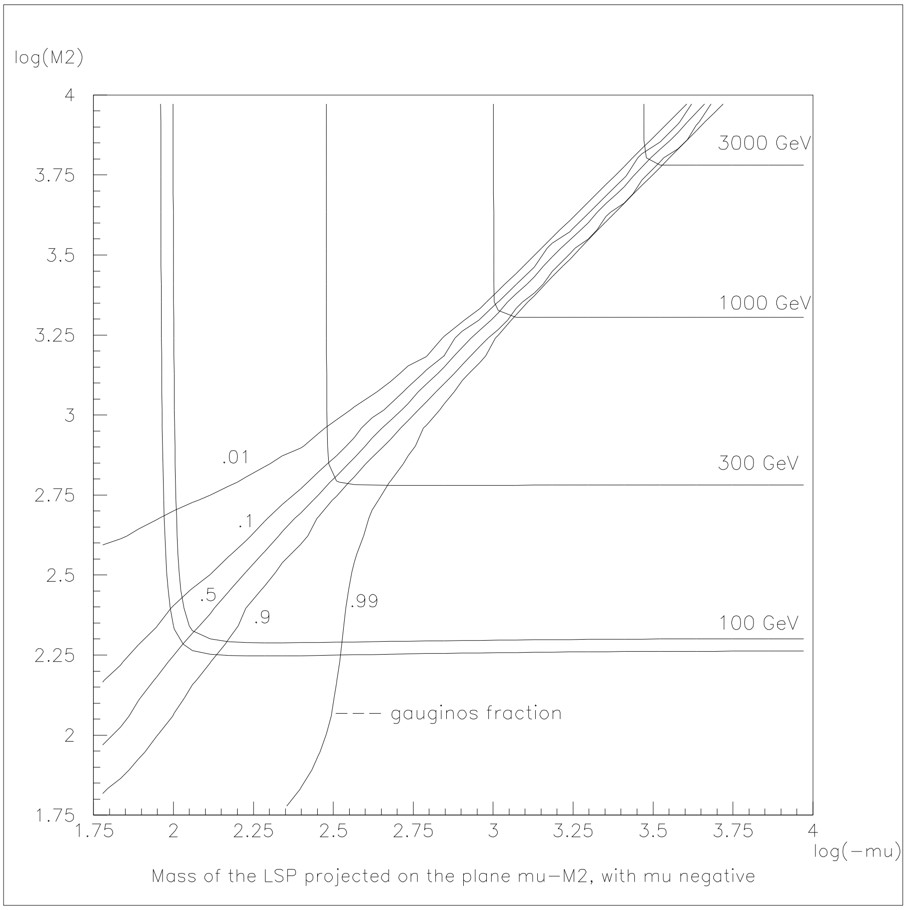

Figure 2 *This figure represents the same projection as in the figure 1 but here μ is negative and varies between $-50\,GeV$ and $-10^4\,GeV$.*

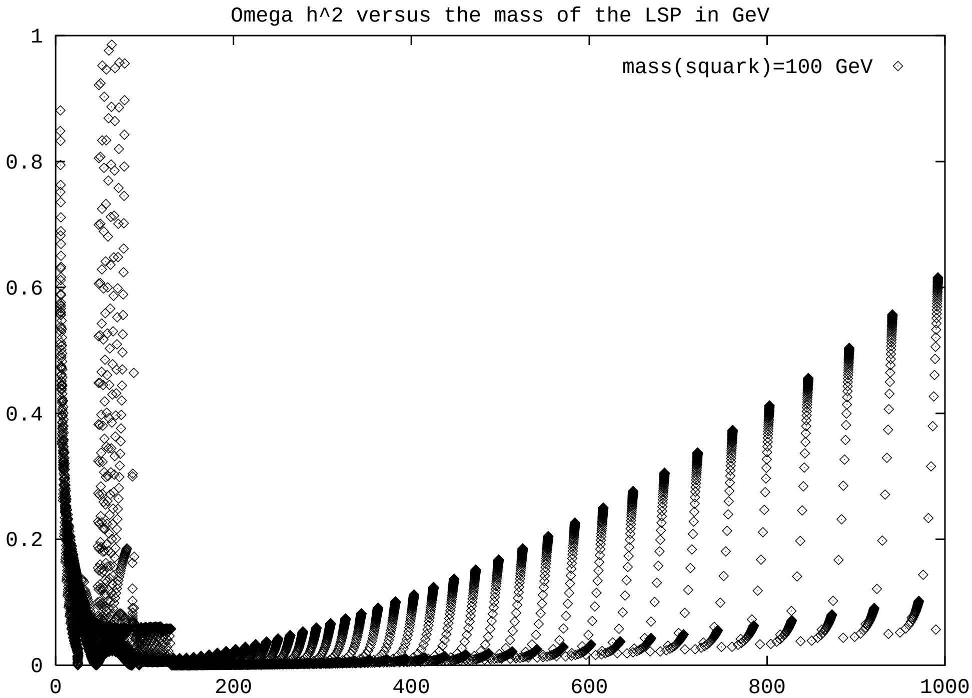

Figure 3 *This figure represents the mass of the LSP versus Ωh^2 (I recall that h is the Hubble's constant divided by 100 $Km\,s^{-1}\,Mpc^{-1}$). This graph was drawn with $\mathrm{tg}\,\beta = 2$, the mass of the lightest Higgs $m_{h^2} = 50\,GeV$, the mass of the Top-quark $m_t = 130\,GeV$ and a mass for the squark of 100 GeV. Ω takes big values when the mass of the LSP is near zero, because of the approximation used in the code but the value of the mass of the squark has an influence on these values. When the mass of the squark is decreased new channels are opened and the LSP can annihilate more, and of course decrease its relic density.*

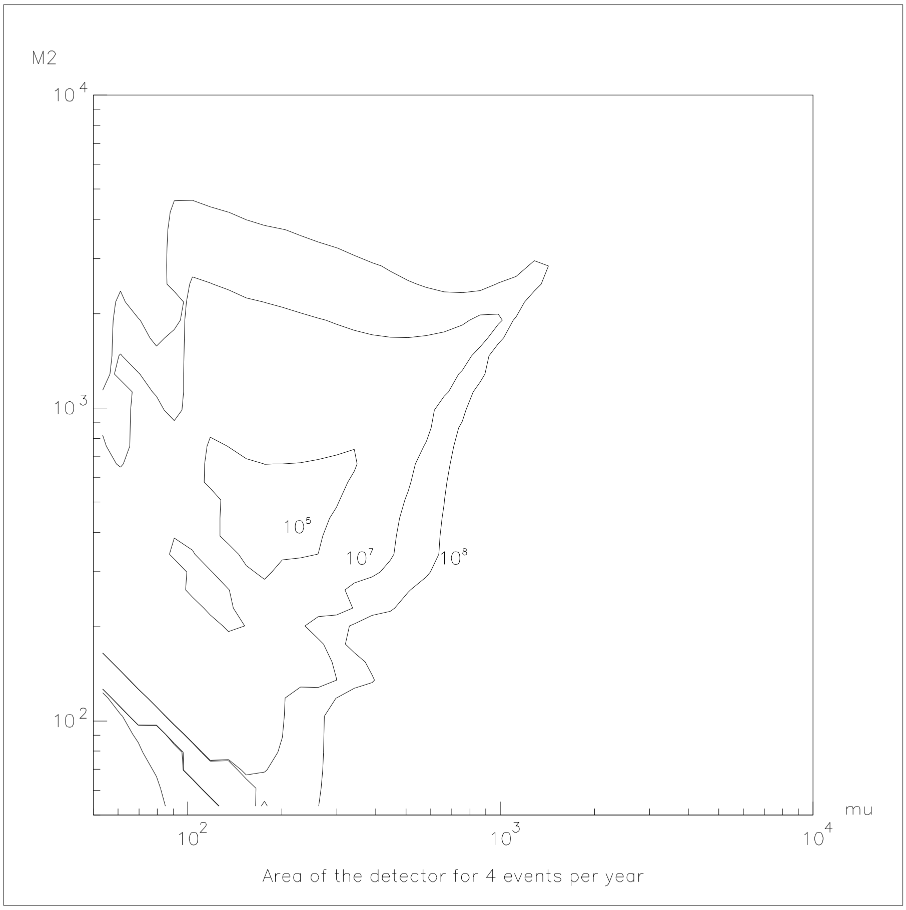

Figure 4 *This figure represents the minimal area in* m^2 *of detector needed to detect 4 events per year. The plot is different of this of the article from Halzen et al. (*HALZEN et al. 1992*) because they have made some mistakes in the program, and the capture rates calculated were greater.*

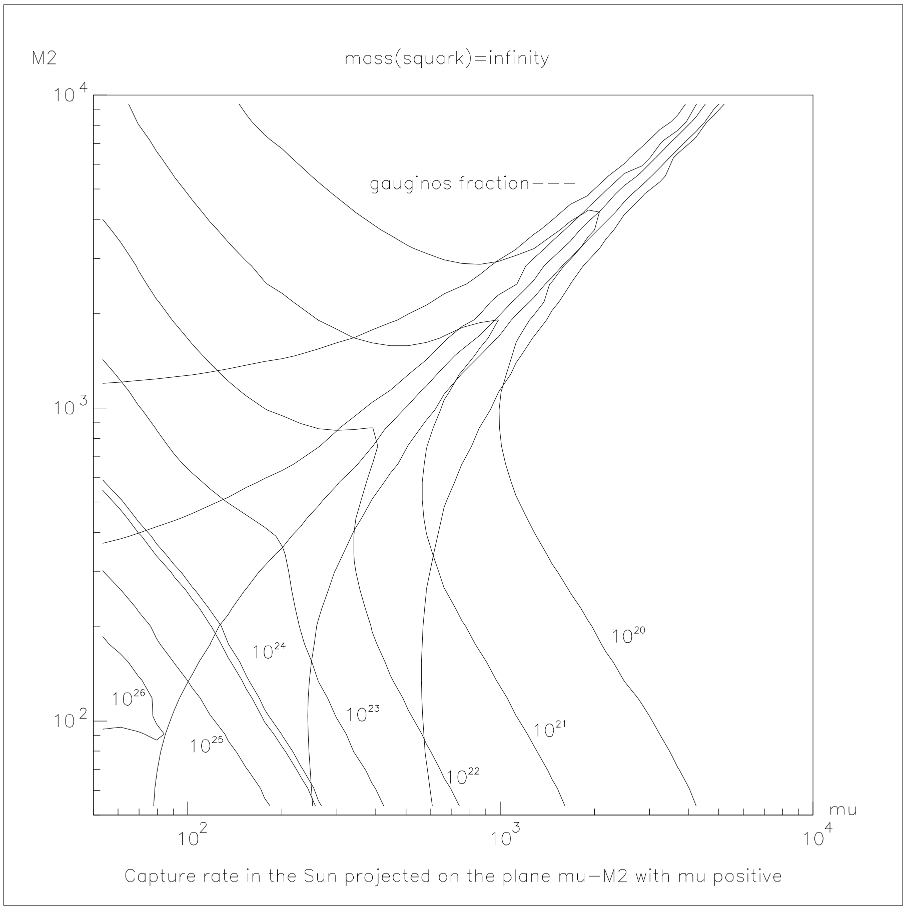

Figure 5 *This figure represents the capture rate in* s^{-1} *of the LSP in the Sun for* $50 \le M_2 \le 10^4\,GeV$, $50 \le \mu \le 10^4\,GeV$, $\mathrm{tg}\,\beta = 2$, *the mass of the lightest Higgs is taken to be equal to 50 GeV and the masses of the squarks taken to be infinite. The double line represent a capture rate of* $10^{24}\,s^{-1}$. *The fraction of gauginos in the neutralino is superposed.*

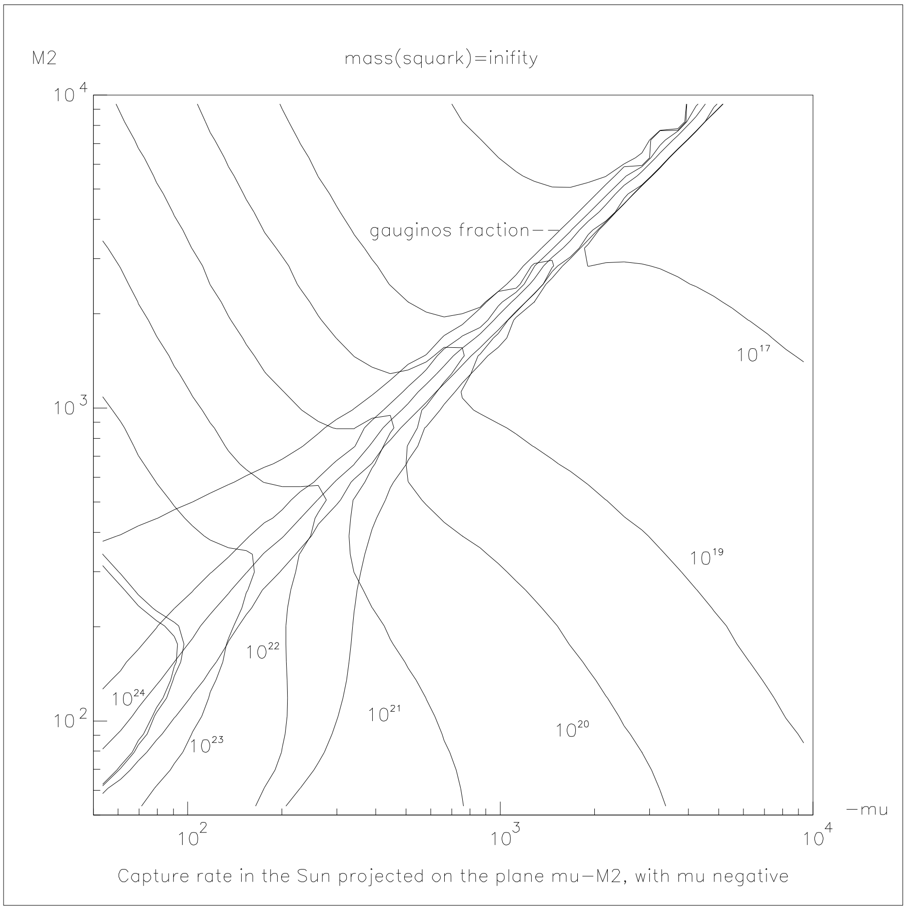

Figure 6 *This figure represents the capture rate in* s^{-1} *of the LSP in the Sun for* $50 \leq M_2 \leq 10^4\,GeV$ *and* $-10^4 \leq \mu \leq -50\,GeV$, $\mathrm{tg}\,\beta = 2$, *the mass of the lightest Higgs is taken to be equal to 50 GeV and the masses of the squarks taken to be infinite. The double line represent a capture rate of* $10^{24}\,s^{-1}$. *The fraction of gauginos in the neutralino is superposed.*

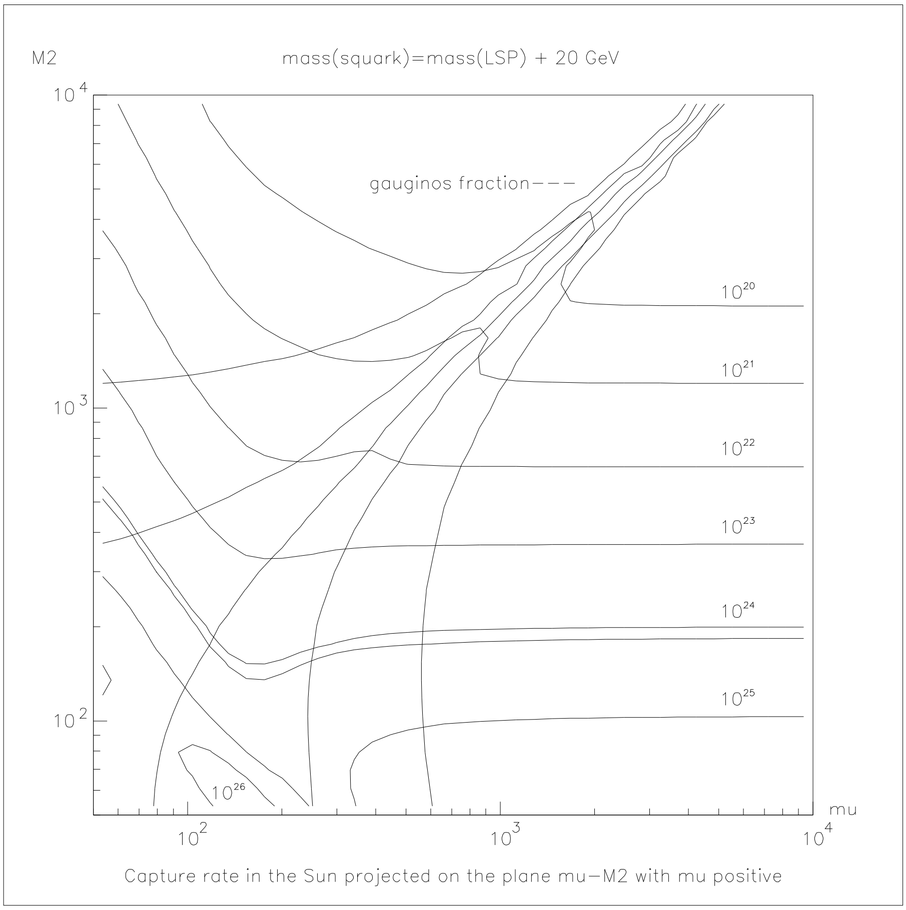

Figure 7 *This figure represents the capture rate in* s^{-1} *of the LSP in the Sun for* $50 \le M_2 \le 10^4\, GeV$, $50 \le \mu \le 10^4\, GeV$, $\mathrm{tg}\,\beta = 2$, *the mass of the lightest Higgs is taken to be equal to 50 GeV and the masses of the squarks 20 GeV greater than the mass of the LSP. The double line represent a capture rate of* $10^{24}\, s^{-1}$.

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Appendix I : Spontaneous symmetry breaking (SSB): the Higgs' mechanism.

I will not give any references in this text, but the interested reader can consult the book from Francis Halzen and Alan D. Martin *Quark & Leptons: an introductory course in modern particle physics* published by Addison Wiley, New York 1984.

I.1 – The Standard Model of particle physics

In 1970 it was understood that the structure group of a particle physics theory *organizes* the different particles and gives the *dynamic* of the theory.

But the lagrangians constructed verifying the invariance under these groups have not a lot of physical meaning, since they give to the theory more symmetries than experienced and massless gauges bosons. For instance, the structure group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ of the Standard Model of particle physics gives long range interactions and zero mass for the $W^{\pm}$ and Z^0, whereas the observations give a range of 10^{-16} cm for the interactions and masses $m_{W^{\pm}} \sim 80$ GeV for the $W^{\pm}$, $m_{Z^0} \sim 91$GeV for the Z^0 (the first experimental evidences of these neutral currents were in 1973 at CERN and FermiLab, it was the great confirmation of this theory). We can cope with these problems by adding a complex doublet field, the Higgs Field, to the lagrangian and performing a spontaneous symmetry breaking.

Because this is a very important idea of the particle physics theory, but without yet any experimental evidences, in section I.3 I will detail the spontaneous symmetry breaking of U(1) and give the result for the spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y$ into the electromagnetism group $U(1)_{EM}$.

I.2 – The theory before spontaneous symmetry breaking

The structure group of the electro-weak theory is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The group $SU(3)_C$ is the color group which classifies the quarks in triplets. The $SU(2)_L$ group deals with the weak processes and the group $U(1)_Y$ with the hyper-charge. The electro-weak lagrangian required by an $SU(2)_L \otimes U(1)_Y$ (I will not speak about the color so I drop the $SU(3)_C$ component) invariance is:

$$\mathcal{L} = \bar{\chi}_L \gamma^\mu \left[i\partial_\mu - g\frac{1}{2}\mathbf{t}\cdot\mathbf{W}_\mu - g'\left(-\frac{1}{2}\right)B_\mu \right]\chi_L + \bar{e}_R\gamma^\mu\left[i\partial_\mu - g'(-1)B_\mu\right]e_R - \frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

where the γ^μ are the Dirac matrices, $\bar{\chi}$ the Dirac conjugaison $\bar{\chi} = \chi^\dagger\gamma^0$, $\mathbf{t} = (\sigma_1, \sigma_2, \sigma_3)$ with σ_i the Pauli's matrices ($\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$), and g and g' are the coupling constants of the gauge fields with the fermions fields.

χ_L stands for any left-handed doublet, *e.g.*, $\binom{\nu}{e^-}_L$ and e_R is the right-handed singlet electron. $\mathbf{W}$ are the weak gauge bosons, they form a SU(2) triplet ($\mathbf{W}^1$,$\mathbf{W}^2$,$\mathbf{W}^3$) and B is the hypercharge gauge boson it lies in a SU(2) singlet. I have inserted the hypercharge values $Y_{L,R}$ for the SU(2) doublet and singlet in the parenthesis before B_μ. The relation $Q \widehat{=} \tau_3 + \frac{Y}{2} = -1$ and that the weak isospin is 1/2 for the SU(2) doublets give $\mathrm{Y}_L = -1$ and the weak isospin for the singlets is 0 give $\mathrm{Y}_R = -2$.

Let have a look at this lagrangian.
The terms between the brackets $D_\mu \widehat{=} [\cdots]$ are the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ covariant derivatives. "$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ covariant derivative" means that if you perform a general $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ transformation this term will be invariant. More precisely, if one imposes the invariance of the lagrangian $\mathcal{L} = \bar{\psi} i \gamma^\mu D_\mu \psi$ under a locale $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge transformation:

$$\begin{array}{lcl} \psi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L & \rightarrow & e^{iY\beta(x) + i\mathbf{a}(x)\cdot\mathbf{T}}\, \psi_L \\ \psi_R = \bar{e}_R & \rightarrow & e^{iY\beta(x)}\, \psi_R \end{array} \qquad (*)$$

where $\mathbf{T}$ is a SU(2) generator of spin-1/2 (the coefficients β and $\mathbf{a}$ are function of x because I have made a *local* transformation, the global transformations are less interesting since they do not need gauge fields).

one finds that:

- There **must** exist two gauge fields B_μ and $\mathbf{W}_\mu$ which transform under $(*)$ by:

$$\begin{array}{lcl} B_\mu & \rightarrow & B_\mu + \frac{1}{g'}\partial_\mu \beta(x) \\ \mathbf{W}_\mu & \rightarrow & \mathbf{W}_\mu - \frac{1}{g}\partial_\mu \mathbf{a} - \mathbf{a} \times \mathbf{W}_\mu \end{array}$$

- The covariant derivative must be:

$$D_\mu = \partial_\mu + ig\mathbf{T}\cdot\mathbf{W}_\mu + ig'\left(\frac{Y}{2}\right) B_\mu.$$

The two last terms in $\mathcal{L}$ are the kinetic energies of the gauge fields with[11] :

$$\begin{aligned} \mathbf{W}_{\mu\nu} &= \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned}$$

A look at this lagrangian shows us that there is *no mass terms* for the gauge bosons, such $-\frac{1}{2}m_B^2 B_\mu B^\mu$, and for the fermions, $-m_F \bar{\psi}\psi$. In fact a term like $-m_e \bar{e} e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$, for the fermions, is forbidden since e_R is a singlet and e_L a doublet.

The Higgs' mechanism for spontaneous symmetry breaking will provide masses for the gauge bosons, therefore a shorter range for the interactions, and masses for the fermions.

[11] The term $W_\mu \times W_\nu$ comes from the non-abelian structure of SU(2).

I.3 – Breaking U(1)

For pedagogy, I detail the breaking of U(1) for a complex scalar field $\phi = \phi_1 + i\phi_2$, which is the same mechanism than for $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ but is mathematically simpler because U(1) has only one generator.

As I explained in the previous section, a local U(1) invariance $\phi \to e^{i\alpha(x)}\,\phi$ gives a covariant derivative $D_\mu = \partial_\mu - ieA_\mu$, where e is the coupling constant between the particle field and the gauge field A_μ, which transforms as $A_\mu \to A_\mu + (1/e)\partial_\mu\alpha$.
So the lagrangian is

$$\mathcal{L} = (D_\mu\phi^*)(D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$. The U(1) invariance forbids terms like $m^2 A_\mu A^\mu$, hence the gauge boson A_μ is massless.
If we add the term

$$+\,\mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

to the lagrangian, we will find that because this potential has a global minimum for $|\phi| = v = \sqrt{\frac{\mu^2}{\lambda}}$ the massless boson A_μ acquires some mass.

If we **choose**, here is the symmetry breaking,

$$\begin{cases} \phi_1^0 = v \\ \phi_2^0 = 0 \end{cases}$$

develop ϕ around this minimum $\phi = \frac{1}{\sqrt{2}}[v + \eta(x) + i\xi(x)]$ and replace in the lagrangian we obtain (writing only the relevant terms for our explanation):

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\xi)^2 + \frac{1}{2}(\partial_\mu\eta)^2 - v^2\lambda\,\eta^2 + \frac{1}{2}\,e^2v^2\,A_\mu A^\mu - ev\,A_\mu\partial^\mu\xi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{interaction terms.}$$

A close look at this lagrangian shows us that appeared:

- A massless boson ξ. It is the so-called Goldstone boson which appears always when a continuous symmetry is broken.
- A massive boson η with mass $\sqrt{2\lambda v^2}$, it is the so-called Higgs' boson.
- A mass term ev for the gauge field A_μ.

This seems good but not quite, because the field A_μ has some mass but has still 2 degrees of freedom instead of 3 as required for a massive field. But we can solve this problem easily if we use the Goldstone boson as the missing longitudinal degree of freedom for A_μ.
If we have a look back to the expression of ϕ we see that we can write

$$\phi = \frac{1}{\sqrt{2}}[v + \eta(x) + i\xi(x)] \simeq \frac{1}{\sqrt{2}}[v + \eta(x)]\,e^{i\xi/v}.$$

Thus taking

$$\phi = \frac{1}{\sqrt{2}}[v + h(x)]\,e^{i\theta(x)/v}$$

and change A_μ into $A_\mu + \dfrac{1}{ev}\partial_\mu\theta$. This gives a longitudinal polarisation to A_μ and the final lagrangian is, including all the interactions terms:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \lambda v^2\, h^2 + \frac{1}{2}e^2 v^2\, A_\mu A^\mu - \lambda v\, h^3 - \frac{1}{4}\lambda\, h^4 + \frac{1}{2}e^2\, A_\mu A^\mu\, h^2 + ve^2\, A_\mu A^\mu\, h. \quad (**)$$

In the previous calculation, we started with a U(1) invariant theory and we broke this invariance, the result is *a mass and a longitudinal polarisation for the gauge boson A_μ and a massive scalar boson the Higgs h.*

I.4 – The electro-weak theory

As explained in section I.1 the electro-weak **must** be broken to cope with the results from the particle colliders. For breaking $SU(2)_L \otimes U(1)_Y$ to $U(1)_{EM}$ we choose a complex Higgs doublet with hypercharge $Y = \pm 1$, so there will be two neutral Higgs and one with charge -1 and one with charge $+1$.
Write the Higgs doublet like:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi + i\phi_3\end{pmatrix} = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}.$$

And add to the lagrangian $\mathcal{L}$ the term:

$$\mathcal{L}_{\text{Higgs}} = (\,D_\mu\phi\,)^\dagger(\,D^\mu\phi\,) - V(|\phi|)$$

Where

$$V(|\phi|) = -m^2\,\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2.$$

The parameters m and λ are arbitrary but the sign of $-m^2 < 0$ is the trigger of the spontaneous symmetry breaking.
This potential has a global minimum for the vacuum expectation value $v.e.v\ |\phi| = \frac{m^2}{2\lambda} \neq 0$. Developping the term $|D_\mu\phi|^2$ around its minimum, one finds two terms like $|\phi|^2 B_\mu B^\mu$ and $|\phi|^2 W_\mu W^\mu$ which give mass to the gauge bosons.

$$\left|\left(-ig\frac{\mathbf{t}}{2}\cdot\mathbf{W}_\mu - i\frac{g'}{2}B_\mu\right)\right|^2 = \left(\frac{vg}{2}\right)^2 W^+_\mu W^{-\mu} + \frac{v^2}{8}(W^3_\mu\ B_\mu)\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{3\mu} \\ B^\mu\end{pmatrix}$$

With $W^\pm_\mu = 1/\sqrt{2}(W^1_\mu \mp iW^2_\mu)$ so there is a mixing of the two first components of $\mathbf{W}_\mu$ in the same way than the neutralino (see section 5.7) is a mixing of the neutral gauginos and neutral higgsinos. These two massives bosons carry some weak charge and are involved in the processes where the weak charge changes.

We can give masses to the fermions with the *same* Higgs doublets by adding a term like

$$\mathcal{L} = -G_e\left[(\bar{\nu}_e\ \bar{e})_L\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} e_R + \bar{e}_R\,(\phi^-\ \bar{\phi}^0)\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L\right]$$

Where G_e is an *arbitrary* constant.

This give the term $-m_e\,\bar{e}e - m_e/v\;\bar{e}e\,h$ where $m_e = vGe/\sqrt{2}$ is the mass of the fermions. In the same way we can give mass to the quarks.

The Higgs' mechanism gives mass to the fermions and the gauge bosons. Now I will use the denomination $Z^0 = -B\sin\theta_w + W^3\cos\theta_w$ and $\gamma = B\cos\theta_w + W^3\sin\theta_w$ instead of W^3 and B. The W have now a mass of $m_W = 1/4g^2\sigma^2$, the Z^0 a mass of $m_Z = 1/4(g^2+g'^2)\sigma^2$ and the photon is still massless $m_\gamma = 0$ and $\sigma^2 = m^2/\lambda = (246\,\mathrm{GeV})^2$ The final electro-weak lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\\ &+\bar{L}\gamma^\mu\Big(i\partial_\mu - g\frac{1}{2}\tau\cdot\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu\Big)L\\ &+\bar{R}\Big(i\partial_\mu - g'\frac{Y}{2}B_\mu\Big)R\\ &+\Big|\Big(i\partial_\mu - g\frac{1}{2}\tau\cdot\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu\Big)\Big|^2 - V(|\phi|)\\ &-(G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + \text{hermitian conjugate})\end{aligned}$$

where L denotes the left-handed fermion (quark and lepton) doublets, R the right-handed fermion singlets and $\phi_c \mathrel{\widehat{=}} 1/\sqrt{2}\begin{pmatrix} v+h \\ 0\end{pmatrix}$.

- The first line of the lagrangian is the $W^\pm$, Z, γ kinetic energies and self-interactions.
- The second and the third line are the lepton and quark kinetic energies and their interactions with the $W^\pm$, Z, γ.
- The fourth line is the $W^\pm$, Z, γ, Higgs masses and couplings.
- The fith line is the lepton and quark masses and coupling with the Higgs.

I.5 – Physical meaning of spontaneous symmetry breaking

In section I.1, we started with a U(1) invariant theory and we broke this invariance. The result was a mass and a longitudinal polarisation for the gauge boson A_μ. Moreover in section I.4 after the breaking of symmetry a new energy scale $\sigma = 246$ GeV appeared. What have been done ?

The theory given by the lagrangian (∗∗) is no longer U(1) invariant since the previous lagrangian is not U(1) invariant. The breaking of the symmetry occurred when we chose a ground state for ϕ.

It is the same phenomena than in the theory of ferro-magnetism where the O(3) symmetry is broken at low temperatures, because the average value of the magnetic moments is not 0, *i.e.*, the magnetic dipoles have "chosen" a special spatial direction. Moreover in the ferro-magnetism theory the O(3) symmetry is restored for temperature above the Curie temperature. Here for the spontaneous symmetry breaking for energies greater than $\sigma = 246$ GeV the complete $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetry is restored.

Appendix II: The decoupling epoch

In this appendix I derive the equation which govern the evolution of the number density of LSP given in the section 6.1 (see the last equation of this appendix).

The interested reader can consult the monography from J. Bernstein *kinetic theory in the expanding universe* Cambridge university press 1988.

At the beggining of the universe, all the different species were in thermal equilibrium. But when the temperature decreased, because of the expansion of the universe, some species came out of equilibrium and were "frozen" in the state they were at this epoch. The problem is to determine the phase space distribution at the epoch of decoupling. Roughly we can quantify the starting time of this epoch by the comparison of the interaction rate by particle Γ and the expansion rate of the universe H. A species is coupled, *i.e.*, is in thermal equilibrium with the other species in the universe, if it interacts a lot with them, that is if its free mean path is greater than the size of the universe. As the free mean path of a particle is proportional to Γ^{-1} and the size of the universe is about H^{-1}, we have

$$\Gamma \gtrsim \mathrm{H} \qquad \text{coupled}$$

$$\Gamma \lesssim \mathrm{H} \qquad \text{decoupled}$$

To solve this problem we can have a look at the phase space distribution for one particle $f(x^\mu, v_\mu)$. The assumed isotropy and homogeneity of the universe allow only phase space distribution depending on the norm of the speed $|v|$ and on the time t, $f(|v|, t)$.

The Boltzmann's equation gives the evolution rate of this distribution. Classically the evolution of the distribution f of a particle in an external field $\mathbf{F}$ *without* interactions with other particles is given by

$$\frac{df}{dt} = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{dt} + \frac{\partial f}{\partial v^\mu}\frac{dv^\mu}{dt} \cdot$$

Since $dx^\mu/dt = v^\mu$ and $dv^\mu/dt = \mathbf{F}/m$ the equation for f is

$$\frac{df}{dt} - \frac{\partial f}{\partial x^\mu}v^\mu + \frac{\mathbf{F}}{m}\frac{\partial f}{\partial v^\mu} = 0.$$

We define the Liouville's operator $\mathbf{L}$ by the left-hand-side of this equation. The right-hand-side of this equation is zero only because we supposed that the particle has no interactions. If there are some interactions with other particles the right-hand-side is given by the collision operator $\mathbf{C}(f)$, which contains all the terms responsible for the variation of f because of the interactions with the other particles.

In general relativity the form of the Liouville's operator is different since it needs to be written in a covariant form. So $\dfrac{d}{dt} + \mathbf{v}\cdot\nabla_x$ becomes $p^\alpha\dfrac{\partial}{\partial x^\alpha}\cdot$ The evolution rate of the impulsion p^μ on the word-line of the particle is $dp^\alpha/d\tau = -m\Gamma^\alpha_{\beta\gamma}\dfrac{dx^\alpha}{d\tau}\dfrac{dx^\beta}{d\tau} = m(-\Gamma^\alpha_{\beta\gamma}v^\beta v^\gamma)$ (we use $m\,dx^\alpha/d\tau = p^\alpha$). So the term in parenthesis $(\cdots)$ is the similar to the force $\mathbf{F}$, thus $\dfrac{\mathbf{F}}{m}\cdot\nabla_v$ becomes $-\Gamma^\alpha_{\beta\gamma}v^\beta v^\gamma\dfrac{\partial}{\partial v^\alpha}\cdot$

Using the results of the Appendix III for the value of the Christoffel's symbols for the Friedmann-Robertson-Walker's cosmology one finds

$$\frac{df}{d\tau} = \frac{df}{dt}v^0 + \mathbf{v}\cdot\nabla f - 2\frac{\dot{R}}{R}v^0 v^i \frac{df}{dv}^i - v^2 \dot{R}R\frac{df}{dv}^0 .$$

Because of the assumed isotropy of the universe, f does not depend on x^i, and then the term $\mathbf{v}\cdot\nabla f$ does not contribute.

In fact this is not the real space-phase distribution of the particle. Because our particle has a mass m we have to add the constraint $1 = v^{0\,2} - v^2$ so the real space-phase distribution is given by

$$\hat{f} = \int f(x^\mu, v^\mu)\,\delta(v^0 - \{1+v^2\}^{1/2})\,dv^0.$$

And the Liouville's operator is re-defined by

$$\mathbf{L}(f) = \int \frac{\partial f}{\partial x^0}\frac{1}{v^0}\,\delta(v^0 - \{1+v^2\}^{1/2})\,dv^0 \tag{5}$$

A straightforward calculus leads to

$$\mathbf{L} = \frac{\partial \hat{f}}{\partial t} - \frac{\dot{R}}{R}p\,\frac{\partial \hat{f}}{\partial p} = \mathbf{C}(f) \tag{6}$$

with $p = \sqrt{\mathbf{p}\cdot\mathbf{p}} = \sqrt{\mathbf{v}\cdot\mathbf{v}}/v^0$

Hereafeter, I will only consider the distribution $\hat{f}$, so I leave the hat and name it f.

If we integrate equation (6)over p^μ and define $n(t)$ the number density of particle by

$$n(t) \,\widehat{=}\, \frac{g}{(2\pi)^3}\int f(E,t)\,d^3p, \tag{7}$$

where g is the number of internal degrees of freedom, *e.g.*, the spin ...,
equation (6) gives

$$\frac{dn}{dt} + 3\frac{\dot{R}}{R}n = \frac{g}{(2\pi)^3}\int \mathbf{C}(f)\,\frac{d^3p}{E}. \tag{8}$$

This equation shows us that there are two causes for the variation of the number density n: the right-hand-side of this equation due to the collisions with the other particles and the second term of the left-hand-side of this equation, due to the expansion of the universe. This term means that even if the particle has no interactions its number density will not be constant (in fact its $R^3 n$ the number density by *co-volume* which is conserved *cf.* Appendix III).

For the process of decoupling, we are interested into the annihilation process

$$\psi + \phi \to X + \bar{X}$$

where ψ is the particle we are studying. In general, ϕ is the anti-particle of ψ, but in the case of the LSP ϕ is equal to ψ.

We define the invariant energy-volume

$$d\Pi_i = \frac{g}{(2\pi)^3}\frac{d^3p_i}{2E_i}.$$

Then the right-hand-side of the Boltzmann's equation is

$$\frac{g}{(2\pi)^3}\int \mathbf{C}(f)\,\frac{d^3p_\psi}{E_\psi} = -\int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar X}\,(2\pi)^4\,\delta^4(p_\psi + p_\phi - p_X - p_{\bar X})$$
$$\Big[\,\big|M(p_\psi, p_\phi; p_X, p_{\bar X})\big|^2 f_\psi f_\phi\,(1 \pm f_X)(1 \pm f_{\bar X})$$
$$-\big|M(p_X, p_{\bar X}; p_\psi, p_\phi)\big|^2 f_X f_{\bar X}\,(1 \pm f_\psi)(1 \pm f_\phi)\Big],$$

where $\big|M(p_\psi, p_\phi; p_X, p_{\bar X})\big|$ is the matrix element for the reaction $\psi + \phi \to X + \bar X$. The first two impulsions in $M(\cdots)$ are for the incoming particles, and the two last impulsions (after the semi-colon) are for the products. The factors $(1 \pm f_X)$ are a consequence of the Pauli's exclusion principle. If the particle X is a fermion, the number of states available for the decay product X are the difference to 1 of the numbers of states already occuped by the others X (because the Pauli's principle allows at most 1 fermion by state), so the number of final states are depleted by a factor $1 - f_X$. This stands for the fermions, for the bosons the "suppression" factor is $1 + f_X$.

Hereafter I assume that the reaction is invariant under *Time reversal* and *Parity transformations*.
By a *time reversal* transformation we change the direction of the time, so we see the reaction $\psi + \phi \to X + \bar X$ running backward. This transformation changes all the impulsions p_i in $-p_i$ (because p_i is a first order time derivative $p_i = m\,dx_i/dt$), so $M(p_\psi, p_\phi; p_X, p_{\bar X})$ is changed in $M'(-p_X, -p_{\bar X}; -p_\psi, -p_\phi)$. Strictly speaking, M' is another matrix of interaction.
Parity transformations are the transformations which change all the processes by their images in a mirror. I consider the transformations that change the vectors into their opposites, x_i is changed into $-x_i$. By this transformation $M(p_\psi, p_\phi; p_X, p_{\bar X})$ is changed into $M''(-p_\psi, -p_\phi; -p_X, -p_{\bar X})$. Strictly speaking, M'' is another matrix of interaction.
I assumed that the process is invariant if we perform these two transformations:

$$M(p_\psi, p_\phi; p_X, p_{\bar X}) \xrightarrow{\textit{time reversal}} M'(-p_\psi, -p_\phi; -p_X, -p_{\bar X}) \xrightarrow{\textit{parity}} M(p_X, p_{\bar X}; p_\psi, p_\phi)$$

Thus I set $M(p_\psi, p_\phi; p_X, p_{\bar X}) = M(p_X, p_{\bar X}; p_\psi, p_\phi) \,\widehat{=}\, M$.

It is well known that this symmetry is not exact in the Nature, since we experiment Parity violation, *e.g.*, with the weak-processes, but it is believed that the symmetry Time-Parity-Charge conjugaison, the three at the same time, is an exact symmetry (it will be tested, soon near the end of the century, in CERN with the creation of anti-hydrogen). In my study the particle, the neutralino, is neutral, but the decay products can carry some charges. We can add the charge conjugaison symmetry, but it is more technical to see its effect on the interaction matrix M.

Assuming that the initial number of ψ is equal to this of ϕ, and X having stronger interactions than ψ, we can suppose the chemical potential of X vanishes, and that X is still at equilibrium. This is the case with the neutralinos whose interactions are electro-weak and the decay products like the quarks which interact by electro-weak processes and by strong processes. Supposing that the particles are not degenerated we can approximate the Fermi's suppression factor $1 \pm f$ by 1, and take the Maxwell's distributions for the particles: $f_i = e^{-E_i/T}$. Therefore the equation (8) for n rewrite

$$\frac{dn}{dt} + 3\frac{\dot{R}}{R}n = -\int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar{X}}\, (2\pi)^4\, \delta^4(p_\psi + p_\phi - p_X - p_{\bar{X}})|M|^2\, [f_\phi f_\psi - f_X f_{\bar{X}}].$$

With the conservation of the energy, guaranted by the δ^4 function, gives $E_\phi + E_\psi = E_X + E_{\bar{X}}$ we have:

$$f_X\, f_{\bar{X}} = e^{-(E_X+E_{\bar{X}})/T} = e^{-E_\phi/T} \times e^{E_\psi/T} \widehat{=} f_\phi^{eq} f_\psi^{eq}$$

with f^{eq} the distribution at equilibrium. Rewriting the evolution equation for n one gets

$$\frac{dn}{dt} + 3\frac{\dot{R}}{R}n = -\int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar{X}}\, (2\pi)^4\, \delta^4(p_\psi + p_\phi - p_X - p_{\bar{X}})|M|^2\, [\, f_\phi f_\psi - f_\phi^{eq} f_\psi^{eq}\,].$$

Looking for small deviation from the equilibrium, I write $f_\phi = f_\phi^{eq} \times (1 + \alpha(t))$ with $\alpha \ll 1$ and define the average cross-section times the relative velocity $|v|$ by

$$\langle\sigma|v|\rangle = (n_\psi^{eq})^{-2} \int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar{X}}\, (2\pi)^4\, \delta^4(p_\psi + p_\phi - p_X - p_{\bar{X}})\, |M|^2\, e^{-E_\phi/T}\, e^{-E_\psi/T}$$

where n_ψ^{eq} is given by the equation (2). Recalling the assumption of no-disymmetry between ψ and ϕ so $n_\psi^{eq} = n_\phi^{eq}$, we have immediatly that

$$\int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar{X}}\, (2\pi)^4\, \delta^4(p_\psi + p_\phi - p_X - p_{\bar{X}})|M|^2\, f_\phi^{eq} f_\psi^{eq} = \langle\sigma|v|\rangle\ (n_\psi^{eq})^2$$

For the remaining term "moving" the factor $1 + \alpha(t)$ from the integral on the impulsion into the integral for n^{eq} yields

$$\begin{aligned}\int d\Pi_\psi\, d\Pi_\phi\, d\Pi_X\, d\Pi_{\bar{X}}\, (2\pi)^4\, \delta^4(p_\psi + p_\phi - p_X - p_{\bar{X}})|M|^2\, f_\phi^{eq}(1+\alpha(t)) f_\psi^{eq}(1+\alpha(t)) &= n_\psi^{eq} n_\phi^{eq}\ \langle\sigma|v|\rangle \\ &= (n_\psi(t))^2 \langle\sigma|v|\rangle.\end{aligned}$$

Thus the equation for the density of ψ (it is the same for the density of ϕ) is

$$\frac{dn}{dt} + 3\frac{\dot{R}}{R}n = -\langle\sigma|v|\rangle[\, n_\psi^2 - (n_\psi^{eq})^2\,]$$

Appendix III : Some useful definitions in the frame of the Friedmann-Robertson-Walker's cosmology

The references for this appendix are the two excellent books from: S. Weinberg 1972 *Gravitation and Cosmology* published by John Wiley & Sons, Inc. and from C. W. Misner, K. S. Thorne and J. A. Wheeler *Gravitation* published by W. H. Freeman, San Franciso 1973.

In this appendix I just give value of the different relevant quantities needed in the main paper, calculated in the frame of the Friedmann-Robertson-Walker's cosmology. I just make a brief introduction to this cosmology and I assume that the reader is familiar with the theory of Special Relativity and General Relativity.

III.6 – About homogeneity, isotropy, time and all the stuff

As indicated by the cosmic micro-wave background (see section 3) there are some observational evidences that the universe is homogeneous and isotropic, at least on a large volume, say 10^8 light-years. There is no evidence that *all* the universe is smooth, but our Hubble-volume, which radius is about $10^{28}h^{-1}$ cm, is believed to beso.

In Special Relativity an homogeneous space can be defined by *a space which is everywhere the same at the same time*. In General Relativity the notion of time is ill-defined, and we might first precise what means *at the same time* in the frame of Special Relativity before giving a definition in the frame of the General Relativity. In Special Relativity two events occur at the same time, for a given observer, if they have the same coordinate x^0 (that is the coordinate with the sign different from the other quantities in the squared interval $d\tau^2 = -dx^{0\,2} + dx^{1\,2} + dx^{2\,2} + dx^{3\,2}$), *i.e.*, they are on the same plane of simultaneity.

In General Relativity, at each point $\mathcal{P}$ in the space-time we can erect a locally inertial Lorentz-frame $\mathcal{R}$, which means that we can locally cancel the effects of gravitation (because we believe in the Principle of Equivalence). In this frame the physics is govern by Special Relativity. We can find a space-like hyper-surface $\mathcal{S}$ passing through $\mathcal{P}$ tangent to the plane of simultaneity $x^0 = 0$ of $\mathcal{R}$. We can do this at each point in the space-time, of course the different Lorentz-frames $\mathcal{R}$ do not mesh, this is the different between Special Relativity and General Relativity. On such a surface all the physical quantities, like the pressure p, the energy-density ρ, the temperature T, the curvature R, ... are the same, this is why we called these surfaces *hyper-surfaces of homogeneity*. We are now ready to give a definition of the homogeneity of the space.

Definition: A space is said to be *homogeneous* if and only if *at each point $\mathcal{P}$ of the space-time passes a space-like hyper-surface of homogeneity*.

The universe is assume to have another propriety: the *isotropy*. Lousely speaking, the universe is isotropic if it looks the same in all directions. It is obvious that this definition needs to be more precise since to an oberver riding an X-ray the universe cannot appear isotropic. It is the same phenomena than when your are riding a bicycle under the rain, the rain seems falling toward us. If we assume, following Friedmann, that the galaxies of the universe are describe by a gas following the expansion of the universe, the so-called cosmological fluid, we have a reference for our definition.

Definition: A space is said *isotropic* if and only if *an observer moving with the cosmological fluid cannot distinguish one of his space direction by any local physical measurements*.

It is easy to see that if a space is isotropic it must be homogeneous too.

It worths to notice that the world line of the observer, in the previous definition, is orthonormal to the hyper-surface of homogeneity. This allows us to define some coordinates: *the co-moving coordinates*. Let an hyper-surface of homogeneity S_I and two observers A and B on this surface. They can be sure that there are on the same surface of homogeneity by measuring some physical quantities and check that they have the same values. On this surface they can define some coordinate (x^1, x^2, x^3). And propagate on their own world line during the proper-time $\Delta\tau$ (before they decided to take the same value of their proper-time on the surface S_I). They are now on another hyper-surface of homogeneity $\mathcal{S}$. But the determinism of the General Relativity (same initial conditions for the Ordinary Differential Equations which govern the evolution of the universe, and same $\Delta\tau$ for both of the observers) assures them that they are again on the same surface. Because the two observers were arbitrary on the surface of homogeneity, these surfaces are surfaces of constant τ, and τ is called the cosmological time and will be noted t. Notice that because their world-line are orthonormal to the hyper-surface of homogeneity the observers evolved at *constant* co-moving coordinates $x^1 = cste$, $x^2 = cste$ and $x^3 = cste$. There are said co-moving with the cosmological fluid composed by the stars, the galaxies, and the various clusters.

III.7 – The Robertson-Walker's metric, and related calculations

In General Relativity the evolution of the universe is described by the evolution of the coefficients of the metric tensor. In 1936 Robertson and Walker have shown that an homogeneous and isotropic universe is describe by the metric

$$(ds)^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\Big(d\theta^2 + \sin^2\theta\, d\phi^2\Big)\right) = -g_{\alpha\beta}\, dx^\alpha dx^\beta \tag{9}$$

With $0 \le r < \infty$, $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. In this formula k, which can be positive $k > 0$, negative $k < 0$ or null, is constrained to the three values $+1,\ 0,\ -1$ by a coordinates rescaling (the consequences of this fact are discussed in the section 2 of this paper).

Now I study the differents geometries of the universe and I show that the scale factor of the universe $R(t)$ can be interpreted as the "radius" of the universe.

- If k is taken equal to 1, the spatial part of the metric (9) describes a 3-dimensional sphere. This can be seen by performing two changes of coordinates. First let

$$r = \sin\chi \qquad 0 \le \chi \le \pi,$$

so the spatial part of (9)

$$d\sigma^2 = R^2(t)\Big[\,\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\Big]$$

becomes

$$d\sigma^2 = R^2(t)\big[\,d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\,\big].$$

Now let

$$\begin{aligned} w &= R(t) \qquad\qquad \cos\chi \\ x &= R(t) \qquad \cos\theta\,\sin\chi \\ y &= R(t)\cos\phi\,\sin\theta\,\sin\chi \\ z &= R(t)\sin\phi\,\sin\theta\,\sin\chi \end{aligned}$$

thus $d\sigma^2$ becomes

$$d\sigma^2 = \big[dw^2 + dx^2 + dy^2 + dz^2\big] \tag{10}$$

with w, x, y, t verifying

$$w^2 + x^2 + y^2 + z^2 = R(t). \tag{11}$$

These two equations show that the spatial geometry of any universe with $k = 1$ is that of a 3-sphere with a radius $R(t)$ (equation (11)) embedded in a four-dimensionnal space $\mathbb{R}^4$ (equation (10)).

□ If $k = 0$ the spatial element of (9) is

$$d\sigma^2 = R^2(t)\big[\,dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,\big] \tag{12}$$

So if we set

$$\Sigma \quad \begin{cases} x = R(t)\,r \qquad \cos\theta \\ y = R(t)\,r\cos\phi\,\sin\theta \\ z = R(t)\,r\sin\phi\,\sin\theta \end{cases}$$

the equation (12)transforms as

$$d\sigma^2 = dx^2 + dy^2 + dz^2$$

with

$$x^2 + y^2 + z^2 = R^2(t)\,r^2.$$

So the spatial geometry of the universe with $k = 0$ is that of $\mathbb{R}^3$ and the transformation (Σ) between (9) and (12) is just the transformation between the cartesian coordinates and the polar coordinates. Here the scale factor $R(t)$ has totally disappeared, and there is no particular interpretation.

□ If $k=-1$ the spatial element of (9) is

$$d\sigma^2 = R^2(t)\left[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{13}$$

If we performe the coordinate transformation

$$\begin{aligned} w &= R(t) \qquad\qquad \operatorname{ch}\chi \\ x &= R(t) \qquad \cos\theta \operatorname{sh}\chi \\ y &= R(t)\cos\phi\cos\theta \operatorname{sh}\chi \\ z &= R(t)\sin\phi\,\cos\theta \operatorname{sh}\chi \end{aligned}$$

with $0 \le \chi < \infty$, we obtain the condition

$$w^2 - x^2 - y^2 - z^2 = R^2(t). \tag{14}$$

And the metric element (13) rewrite as

$$d\sigma^2 = -dw^2 + dx^2 + dy^2 + dz^2. \tag{15}$$

Thus a universe with $k=-1$ cannot be embedded in an Euclidian space, but in the Minkowski's space $\mathcal{M}_4$ (see equation (15)) and the equation (14) shows us that this universe is negatively curvated (like a saddle). The curvature of this space is given by $R(t)$ so we can see it as the radius of the universe (actually $-R(t)$ is the radius of the universe).

Before leaving this quick study of the geometry of the universe, we should emphasis that the *global* geometry is *arbitrary* since the metric (1) gives only the *local* properties. For instance, in the case $k=0$ instead of taking x, y and z varying between 0 and ∞ we could take x, y and z varying between, say, 0 and L with periodical conditions. In this case the universe will be donough. I recall to the reader that a donough is *flat*, since it is homeomorphic to a square with edges identified, the *apparent* curvature is only due to the embedding in $\mathbb{R}^3$.

Now I want to give, some useful definitions and relations:

- On the spatial part of the universe we define rthe elementary (canonical) volume by

$$d^3\mathcal{V} = R^3(t)\frac{r^2\sin\theta}{\sqrt{1-kr^2}}\, dr\wedge d\theta\wedge d\phi$$

  As indicated by the notation $R(t)$, the radius of the universe can vary with the (cosmological) time t, so the elementary volume $d^3\mathcal{V}$ varies in the same way. But in physics we generally define some quantities by unit of volume, like the number density of energy in the universe ρ, and we want that these quantities are conserved. Unfortunatly because of the factor $R(t)$ the relevant volume element is not $d^3\mathcal{V}$ but *the differential co-volume* defined by $d^3\mathcal{V}/R^3(t)$. For example, if n_G is the number density of galaxies in the universe, because the expansion of the universe n_G is not conserved but $n_G R^3(t)$ is.

- I want know to give more explanation about the definition of the starting time $t = 0$ of the universe, given in section 2. The observations of a redshift for the spectral lines of the stars, give that the universe is expanding, thus $H_0 = \dot{R}/R$ is positive. The measurements of the deceleration parameter, $q_0 = -\ddot{R}\,R/\dot{R}^2$, which give q_0 is *positive* too (see WEINBERG 1972), hencefore $\dot{R} > 0$ and $\ddot{R} < 0$. Hence as the only evolutions for R (when $q_0 > 0$) are a concave always growing (unbound open universe) or increasing and decreasing but always as a concave function (close universe) there **must** be one time in the past t_0 when $R(t_0)$ vanished. We define the starting time of the universe $t = 0$ by *the first time in the past before our epoch when R was zero.*
- I give the Christofell's symbols for the metric (1):

$$\Gamma^{0}_{i\,j} = \frac{\dot{R}}{R}\,g_{ij}$$
$$\Gamma^{i}_{0\,j} = \frac{\dot{R}}{R}\,\delta^i_j$$
$$\Gamma^{i}_{j\,k} = \frac{1}{2}\,g^{i\,\ell}\left(\frac{\partial g_{j\,\ell}}{\partial x^k} + \frac{\partial g_{k\,\ell}}{\partial x^j} - \frac{\partial g_{j\,k}}{\partial x^\ell}\right)$$

where i, j and ℓ run between 1 and 3, and δ^i_j is the Kronecker's symbol

$$\delta^i_j = \begin{cases} 0 & \text{if } i \text{ is different from } j \\ 1 & \text{if } i = j \end{cases}.$$

- The Riemann's tensor $\mathbf{R}$ and the Ricci's scalar $\mathbf{R}^{\alpha}{}_{\alpha} = \mathcal{R}$ are

$$\mathbf{R}_{00} = -3\frac{\dot{R}}{R}$$
$$\mathbf{R}_{ij} = -\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R} + 2\frac{k}{R^2}\right)g_{ij}$$

and

$$\mathcal{R} = -6\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R} + 2\frac{k}{R^2}\right)$$

- Now I will speak about the value of Ω expressed in term of the energy density, and the curvature parameter k.

The conservation of the energy in any volume were there is no source nor well (destruction of energy) leads to the cancellation of the divergence of the energy-momentum tensor $\mathbf{T}$

$$\mathbf{T}^{\mu\nu}{}_{;\nu} = 0. \tag{16}$$

As we describe the universe by a perfect fluid with pressure p and energy density ρ the energy-momentum tensor is[12]

$$T_{\mu\nu} = p\,g_{\mu\nu} + (p+\rho)\,U_\mu U_\nu$$

[12] In fact we *do not* have to suppose that because in any homogeneous and isotropic universe the energy-momentum tensor takes the form of which for a perfect fluid (see S. Weinberg 1972 *Gravitation and Cosmology* published by John Wiley & Sons, Inc.)

Where U_μ is a kind of "four-velocity". The equation evolution rate for $R(t)$ deduced from the Einstein's equation

$$\mathbf{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = -8\pi G \mathbf{T}_{\mu\nu}$$

is

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\rho - \frac{k}{R^2}$$

where G is the gravitational constant and ρ the mass-energy density. If we reexpress this equation is term of $\Omega = \rho/\rho_c$ (recall that $\rho_c \,\widehat{=}\, 3\mathrm{H}_0^2/8\pi G$) we have

$$\frac{\Omega - 1}{\Omega} = \frac{3k}{8\pi G \rho R^2}$$

On this formula we can see that $\Omega = 1$ is proportional to k (see section 2. of the paper for a discussion of this fact).
The equation (16) gives

$$R^3 \frac{dp}{dt} = \frac{d}{dt}\left\{ R^3 [\, \rho + p \,] \right\} \tag{17}$$

In the case of an universe dominated by the matter (that is our universe at the present epoch) the pressure is negligible in comparison of the energy density so the equation (17) rewrites as $\rho R^3 = cste$, this means that the energy density by co-volume is constant and $(\Omega - 1)/\Omega \propto R$.
The universe was not always dominated by the matter, the matter domination epoch starts when the temperature dropped below the rest-mass of hydrogen and helium, *i.e.*, T between 10^3–10^5 K or at a time $t \sim 10^5$ years. At this time the electrons and the protons can recombine into atoms. Before this time the radiation was dominating. In an universe dominated by the radiation the pressure is related to the energy density by $p = \rho/3$, so the equation (17) gives that ρR^4 is conserved. This gives that $(\Omega - 1)/\Omega$ is proportionnal to R^2. For a discussion on the implication of this fact on the problem of the dark matter, see section 2 of the paper.