The symphony of gravity

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Séminaire Poincaré

Quantum Gravity

IHP, Paris, France,

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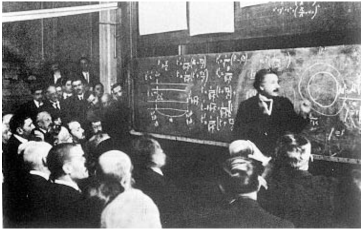
Gravity

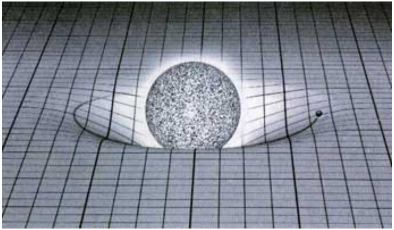
Deux forces règnent sur l'univers :
lumière et pesanteur

Simone Weil, "La pesanteur et la grâce"

Einstein's theory of Gravity

Novembre 1915, Albert Einstein published his theory of gravity:
the general relativity



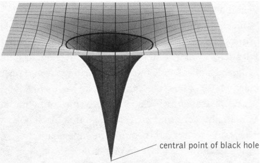


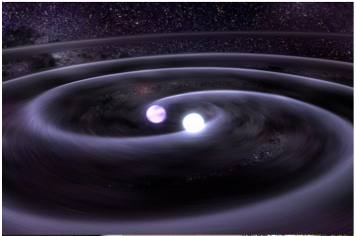
Space and time are dynamical actors of the force of gravity
Gravity is the consequence of the deformation of space and time. A body is not attracted but follows a natural free movement in a curved space-time

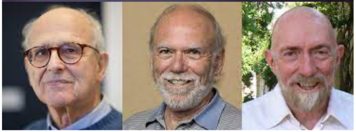
This has deeply changed our vision of space and time

Two fundamental consequences of Einstein's theory of gravity:

Black holes and Gravitational waves



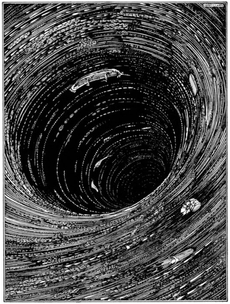




Roger Penrose 2020
"for the discovery that black hole formation is a robust prediction of the general theory of relativity"

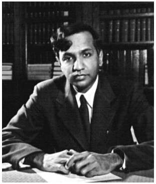
Rainer Weiss, Barry Barish and Kip Thorne 2017
"for decisive contributions to the LIGO detector and the observation of gravitational waves"

Black Holes



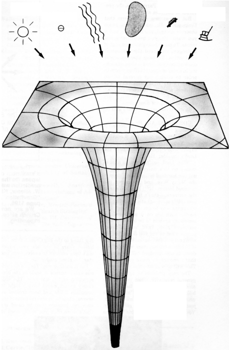
Les physiciens disent des trous noirs qu'à force de se concentrer dans le ciel nocturne, il leur arrive d'enrouler, dans la substance ténébreuse, l'espace qu'ils épanchent dans le temps.

Pascal Quignard
(La barque silencieuse Chap XXV Extase et enstase)



The most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time (S. Chandrasekhar)

No hair theorem for black holes



Characterized by

- Mass M
- Angular momentum $\vec{J}$
- Electric charge $\vec{Q}$

Swallow everything: matter and energy

- We cannot shield their attraction

Singular solution hidden by an event horizon (Cosmic censorship hypothesis, Penrose, Nobel 2020)

In 1939, Einstein argued that black holes are incompatibles with the physical reality of his theory of gravitation.

ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES

By Albert Einstein

(Received May 10, 1939)

If one considers Schwarzschild's solution of the static gravitational field of spherical symmetry

$$ds^2 = -\left(1 + \frac{\mu}{2r}\right)^4 (dx_1^2 + dx_2^2 + dx_3^2) + \left(\frac{1 - \frac{\mu}{2r}}{1 + \frac{\mu}{2r}}\right)^2 dt^2 \quad (1)$$

sents the gravitating mass.)

There arises the question whether it is possible to build up a field containing such singularities with the help of actual gravitating masses, or whether such regions with vanishing g_{44} do not exist in cases which have physical reality. Schwarzschild himself investigated the gravitational field which is produced by an incompressible liquid. He found that in this case, too, there appears a region with vanishing g_{44} if only, with given density of the liquid, the radius of the field-producing sphere is chosen large enough.

This argument, however, is not convincing; the concept of an incompressible liquid is not compatible with relativity theory as elastic waves would have to travel with infinite velocity. It would be necessary, therefore, to introduce a compressible liquid whose equation of state excludes the possibility of sound

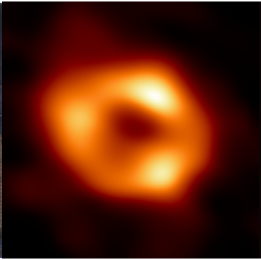
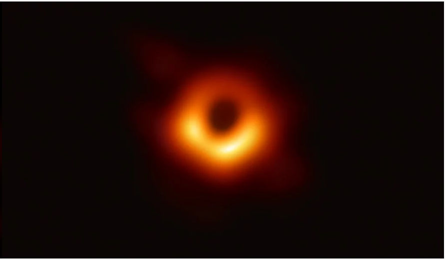
Annal of Mathematics 40 4 (1939) 922-936

- 🥺 Is the singularity of black holes real or a mathematical artefact?
- 🥺 How can matter create a black hole?

In the 1930s, Oppenheimer and Wheeler showed that black holes are a natural consequences of Einstein's theory of general relativity. However, their work was not widely accepted at the time.

It was not until the 1970s that the first observational evidence for black holes was discovered : Sagittarius A*

Seeing Black Holes

- 🎉 In 2019 "Event Horizon Telescope" pictured the supermassive black holes Sagittarius A* and Messier 87*
- ✔ Image first computed in 1979 by Jean-Pierre Luminet
- ✔ The black hole Messier 87* is rotating black hole
- ✔ SgrA* seems to be slowly rotating

How many black holes in the Universe?

SCIENCE NEWS LETTER *for January 18, 1964*

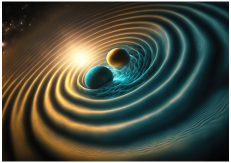
ASTRONOMY

"Black Holes" in Space

Although Einstein doubted the reality of black holes, many direct and indirect detections confirm their presence in our observable universe.

- More than 100 million solar mass black holes in our galaxy
- At least 100 billion supermassive black holes (millions or billions of solar masses) in the universe.
- The largest black hole is in the galaxy NGC4889: its mass is 21 billion solar masses.
- The closest known black hole known as 1A 0620-00 is 3,500 light-years from Earth. The black hole has about 10 times the mass of the Sun.

Gravitational waves



If you ask me whether there are gravitational waves or not, I must answer that I don't know. But it is a highly interesting problem.
(Albert Einstein, 1936)

Gravitational waves

In his 1095 article "Sur la dynamique de l'électron" Henri Poincaré understood that if gravitational interaction is not instantaneous that implies the emission of **ondes gravifiques**

(...) j'ai été d'abord conduit à supposer que la propagation de la gravitation n'est pas instantanée, mais se fait avec la vitesse de la lumière. (...) Quand nous parlerons donc de la position ou de la vitesse du corps attirant, il s'agira de cette position ou de cette vitesse à l'instant où l'**onde gravifique** est partie de ce corps; (...)

He never made this more precise and one had to wait for Einstein's gravity for a proper definition of gravitational waves

Gravitational Waves

In 1936, in an article entitled "Do Gravitational Waves Exist?", Einstein and Rosen argued **against** the existence of gravitational of waves. This article rejected by Physical Review appeared in the Journal of the Franklin Institute under the title "On Gravitational Waves".

ON GRAVITATIONAL WAVES.

BY

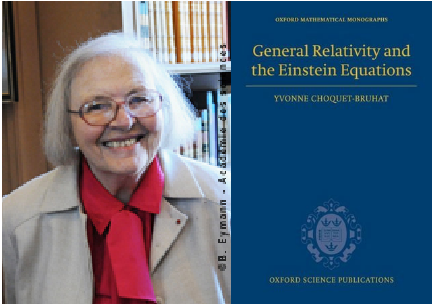
A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

They wondered worried that gravitational waves could not exist and that they would not describe fluctuations of space-time ?

Y. Choquet-Bruhat (29 decembre 1923 -)



On voyait ainsi apparaître des **ondes et rayons gravifiques,** donnant au champ de gravitation le caractère d'un phénomène de propagation et on constatait l'identité entre les lois de propagation de la lumière et du champ de gravitation. (Acta Math. 88: 141-225 (1952))

Y. Choquet-Bruhat gave the first rigorous mathematical proof of the existence of gravitational waves in Einstein theory

Y. Choquet-Bruhat (29 decembre 1923 -)

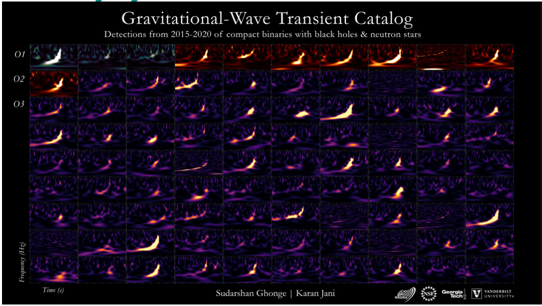
Her results are of great importance for gravitational wave detectors

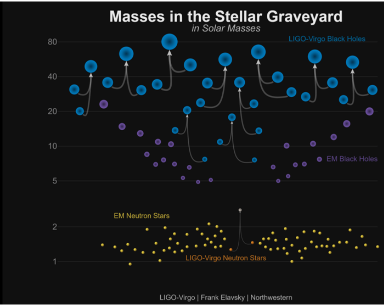
- **Her local theorem of 1952 was a breakthrough** and has since been fundamental for further investigations of the Cauchy problem and proved crucial to the possibility of numerically simulating the motion and gravitational radiation of coalescing binary black holes.
- **She introduced new formulations of Einstein's theory of gravitation** that recently spurred great progress in numerical relativity, including the calculation of the gravitational waves emitted when two black holes collapse and merge together.

Day in Honor of Yvonne Choquet-Bruhat's 100th Birthday (IHES)

Hearing Gravitational Waves

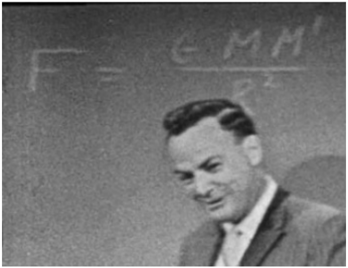
Since September 14, 2015, the LIGO/Virgo collaborations have detected 90 signals of gravitational waves from mergers of binary systems





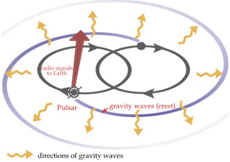
- 🎉 Detection of the dynamics of black holes
- ✔️ Extend the spectrum of detected black holes (from 6 to 40 solar masses)
- ✔️ With the various improvements and developments of ground and future space detectors we expect one detection par week

Quantum Gravity



My subject is the quantum theory of gravity. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; . . .
(Feynman Jablonna, 1962)

Post-Minkowskian expansion for the binary system



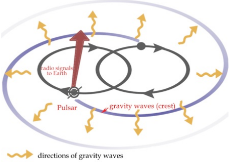
The relativistic two body dynamics can be described by a centre-of-mass Hamiltonian

$$\mathcal{H}_{\rm PM}(p,r) = \sqrt{p^2+m_1^2} + \sqrt{p^2+m_2^2} + \sum_{L\geq 0} \underbrace{\mathcal{V}_{L+1}(p_1\cdot p_2, r)}_{\propto \frac{G_N^{L+1}}{r^{L+1}}}$$

with a relativistic potential organised in a series of Newton's constant G_N which is the general relativity correction to Newton's potential $L=0$

$$V_1(\gamma, r) = -\frac{G_N}{E_1 E_2}\frac{m_1^2 m_2^2}{r}\left(2\gamma^2-1\right) \qquad \gamma = \frac{p_1\cdot p_2}{m_1 m_2} = \frac{1}{\sqrt{1-\frac{\vec{v}^2}{c^2}}} \geq 1$$

Post-Minkowskian expansion for the binary system



The $L+1$PM potential has polynomial mass dependence

$$\mathcal{V}_{L+1}(\gamma, r) = \frac{G_N^{L+1} m_1^2 m_2^2}{r^{L+1}} \sum_{r_1+r_2=L} v_{r_1,r_2}(\gamma)\, m_1^{r_1} m_2^{r_2}$$

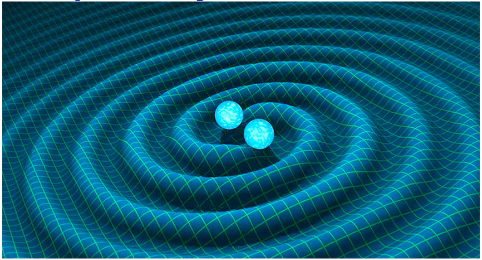
Consider its Fourier transform to momentum space

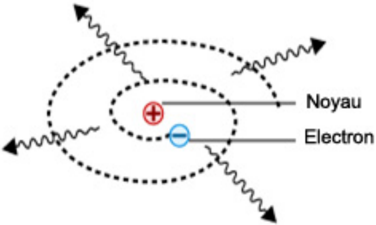
$$\int e^{-i\underline{q}\cdot\vec{r}} \frac{d^3 r}{r^{L+1}} \propto \frac{1}{\underline{q}^{2-L}}$$

leading to

$$\mathcal{M}_L(\gamma, q^2) = \frac{G_N^{L+1} m_1^2 m_2^2}{\underline{q}^{2-L}} \sum_{r_1+r_2=L} v_{r_1,r_2}(\gamma)\, m_1^{r_1} m_2^{r_2}$$

Gravity and Quantum mechanics



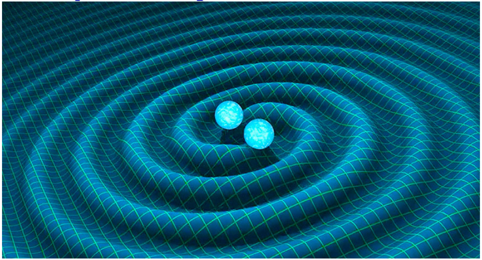


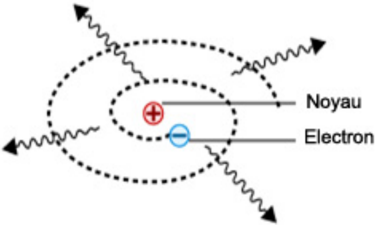
In 1916, Einstein was already arguing for a modification of his theory of gravity by quantum mechanics.

However, according to the inner-atomic electron movement, atoms would have to emit not only electromagnetic, but also gravitational energy, even if in a tiny amount.

Since this is unlikely to be the case in nature, it seems that **quantum theory** will have to modify not only Maxwell's electrodynamics, but also **the new gravitational theory.**

Gravity and Quantum mechanics





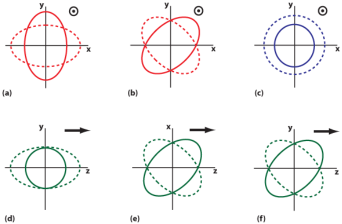
In his famous 1918 paper where he computes the quadrupole formula of emission of classical gravitational waves

> (...) the emission cannot turn negative in any direction; consequently, the total emission certainly cannot turn negative, either. (...) – **which would require a loss of energy of bodies due to the thermal agitation–must raise doubts to the general validity of the theory.**
>
> It seems that a more complete **quantum theory** would also have to bring about a **modification** of the theory of gravitation.

Graviton

In a similar way as electro-magnetic forces are mediated by the photon, quantum gravity assumes that the force is mediated by the graviton a massless particle of spin 2

$$h: \qquad \epsilon_{\mu\nu}^{++}, \qquad \epsilon_{\mu\nu}^{--}$$



Classical physics from quantum loops

THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*†‡

SÁNDOR J. KOVÁCS, JR.

AND

KIP S. THORNE

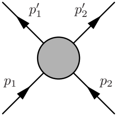
Received 1977 October 21; accepted 1978 February 28

g) The Feynman-Diagram Approach

Any classical problem can be solved quantum-mechanically; and sometimes the quantum solution is easier than the classical. There is an extensive literature on the Feynman-diagram, quantum-mechanical treatment of gravitational bremsstrahlung radiation (e.g., Feynman 1961, 1963; Barker, Gupta, and Kaskas 1969; Barker and Gupta

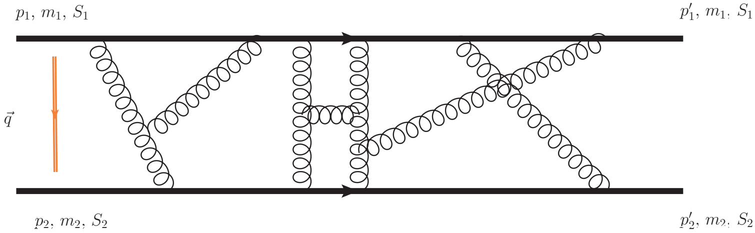
We seek quantum gravity formalism where the classical limit $\hbar \to 0$ gives the general relativity potential

$$\lim_{``\hbar \to 0''}$$



$$\to \mathcal{M}_L(\gamma, q^2) \to \mathcal{V}_{\mathrm{L+1-PM}}(p, r)$$

Classical physics from quantum loops

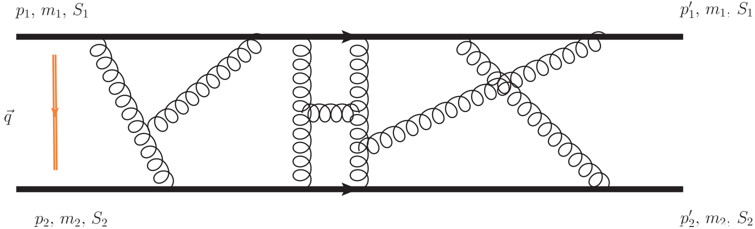


Considering quantized massive fields interacting gravitationally by exchanging massless gravitons and remembering that the QFT propagator has inverse $\hbar$ that the traditional counting disregards

To find the connection between L and the power of $\hbar$, we collect all factors $\hbar$. We leave aside the factor $\hbar$ that gives the mass term a correct dimension. In other words, the Klein-Gordon equation should read $[\partial_x^2 + (mc/\hbar)^2]\varphi = 0$, indicating that the mass term is of quantum origin. This phenomenon is disregarded in the sequel. There are thus two origins of such factors. First the

[Itzykson & Zuber "Quantum Field Theory", §6-2-1 page 288]

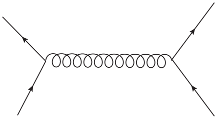
Classical physics from quantum loops



Considering quantized massive fields interacting gravitationally by exchanging massless gravitons and remembering that the QFT propagator has inverse $\hbar$ that the traditional counting disregards the classical potential emerges a piece of a quantum amplitude

$$\mathfrak{M}^L\Big|_{\text{classical}} \propto \frac{m_1^2 m_2^2}{\underline{q}^{2+\frac{(2-D)L}{2}}} \hbar^{L-1} G_N^{L+1} \sum_{r_1+r_2=L} \left(\frac{m_1 c}{\hbar}\right)^{r_1} \left(\frac{m_2 c}{\hbar}\right)^{r_2} \propto \frac{\mathcal{M}_L(\gamma, \underline{q}^2)}{\hbar}$$

One graviton exchange : tree-level amplitude



$$\mathfrak{M}_0 = -16\pi G_N \hbar \frac{2(p_1 \cdot p_2)^2 - m_1^2 m_2^2 - |\hbar \vec{\underline{q}}|^2 (p_1 \cdot p_2)}{|\hbar \vec{\underline{q}}|^2}$$

The $\hbar$ expansion of the tree-level amplitude

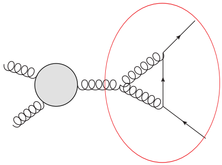
$$\mathfrak{M}_0 = \frac{\mathcal{M}_1^{(-1)}(p_1 \cdot p_2)}{\hbar |\underline{q}|^2} + \hbar 4\pi G_N p_1 \cdot p_2$$

The relativistic classical Newtonian potential is obtained by taking the Fourier transform

$$\mathcal{V}_1(p_1 \cdot p_2, r) = \int \frac{d^3 \vec{\underline{q}}}{(2\pi)^3} \frac{\mathcal{M}_1^{(-1)}(\vec{q})\, e^{i \vec{\underline{q}} \cdot \vec{r}}}{4 E_1 E_2} = -\frac{G_N m_1^2 m_2^2}{E_1 E_2} \frac{2\gamma - 1}{r}$$

The contribution of order $\hbar$ is the quantum contact interaction

Classical physics from loops : the one-loop triangle



Remembering the $\hbar$ in the Klein-Gordon equation

$$(\Box - \frac{m^2c^2}{\hbar^2})\phi = 0$$

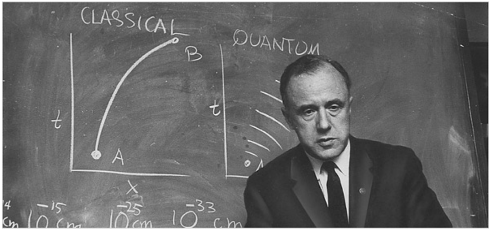
The large mass expansion $\underline{q} \ll m$ of the triangle with a massive leg $p_1^2 = p_2^2 = m^2$ reads

$$\int \frac{G_N\, d^4\ell}{(\ell+p_1)^2(\ell^2 - \frac{m^2c^2}{\hbar^2})(\ell - p_1')^2}\Bigg|_{\text{finite part}} \sim \frac{G_N}{m^2}\left(\frac{\pi^2 mc}{\hbar|\underline{q}|} + \log(q^2) + \cdots\right)$$

This one-loop amplitude contains

- ✔️ The classical 2nd post-Minkowskian correction G_N^2/r^2 to Newton's potential of order $1/\hbar$
- 🕵️ An infrared quantum correction of order $\hbar^0$

Classical gravity from quantum scattering



The *classical* limit $\hbar \to 0$ fixed $\underline{q} \ll m_1, m_2$ of the amplitude

$$\mathfrak{M}_L(\gamma, \underline{q}^2, \hbar) = \frac{\mathcal{M}_L^{(-L-1)}(\gamma, \underline{q}^2)}{\hbar^{L+1}|\underline{q}|^{\frac{L(4-D)}{2}+2}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma, \underline{q}^2)}{\hbar|\underline{q}|^{\frac{L(4-D)}{2}+2-L}} + O(\hbar^0)$$

- ✔ A classical contribution of order $1/\hbar$ from all loop orders
- ✔ The dimensional regularisation scheme gives a control of the IR divergences from radiation
- ✔ Efficient use of modern amplitudes methods and evaluation of Feynman integrals

The connection between quantum scattering and classical gravitational physics has forced to rethink the S matrix for dealing with the $\hbar$ expansion

$$\widehat{S} = \mathbb{I} + \frac{i}{\hbar}\widehat{T} = \exp\left(\frac{i\widehat{N}}{\hbar}\right); \qquad \hat{N} = \sum_{L\geq 0} G_N^{L+1}\hat{N}_L^{\text{classical}} + O(\hbar)$$

$$\mathfrak{M}(\gamma, \underline{q}^2, \hbar) = \sum_{L\geq 0} G_N^{L+1}\left(\frac{\mathcal{M}_L^{(-L-1)}(\gamma, \underline{q}^2)}{\hbar^{L+1}|\underline{q}|^{\frac{L(4-D)}{2}+2}} + \cdots + \frac{\mathcal{M}_L^{(-1)}(\gamma, \underline{q}^2)}{\hbar|\underline{q}|^{\frac{L(4-D)}{2}+2-L}} + O(\hbar^0)\right)$$

- ✔ The higher $1/\hbar$ powers **are needed for the consistency of the full quantum amplitude and the correct exponentiation of the amplitude**
- ✔ One shows that in the conservative sector the v.e.v. $N(\gamma, J) = \langle p_1, p_2|\hat{N}|p_1', p_2'\rangle$ is the radial action from classical Hamilton-Jacobi equation applied to classical GR [Landau, Lifshitz]

$$\chi = -\frac{\partial N(\gamma, J)}{\partial J}$$

Classical Gravity from quantum Gravity

PHYSICAL REVIEW D VOLUME 7, NUMBER 8 15 APRIL 1973

Quantum Tree Graphs and the Schwarzschild Solution

M. J. Duff*
Physics Department, Imperial College, London SW7, England
(Received 7 July 1972)

I. INTRODUCTION

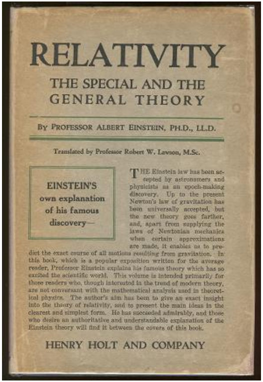
In an attempt to find quantum corrections to solutions of Einstein's equations, the question naturally arises as to whether the $\hbar \to 0$ limit of the quantum theory correctly reproduces the classical results. Formally, at least, the correspondence between the tree-graph approximation to quantum field theory and the classical solution of the field equations is well known,[1] i.e., the classical field produced by an external source serves as the generating functional for the connected Green's functions in the tree approximation, the closed-loop contributions vanishing in the limit $\hbar \to 0$. The purpose of this paper is to present an explicit calculation of the vacuum expectation value (VEV) of the gravitational field in the presence of a spherically symmetric source and verify, to second order in perturbation theory, that the result is in agreement with the classical Schwarzschild solution of the Einstein equations. This would appear to be the first step towards tackling the much more ambitious program of including the radiative quantum corrections.

In 1973 Duff analyzed the question of the classical limit of quantum gravity by extracting the Schwarzschild back hole metric from quantum tree graphs

This is an important consistency check that we understand how to embed Einstein gravity into quantum gravity

- ✔️ We can now derive various black-hole solutions in four and higher dimensions to high orders in G_N
- 🕵️ The framework gives quantum corrections to the classical geometry

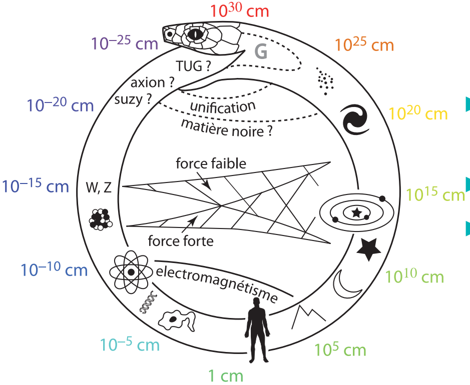
Gravity as an effective field theory



The most beautiful fate of a physical theory is to point the way to the establishment of a more inclusive theory, in which it lives on as a limiting case.

Albert Einstein, “Relativity: The Special and the General Theory” (1916)

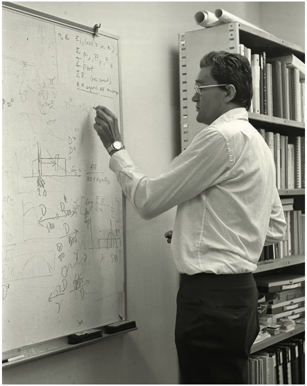
The force of Gravity is universal



- Act on all kind of matter and energy
- Always attractive
- Acts at all scales from microscopical scales $10^{-35}m$ till $10^{26}m$

Einstein's theory of gravity is main framework for analysing our **Universe** from **small** scales to **cosmological** scales

Quantum gravity : the best effective field theory

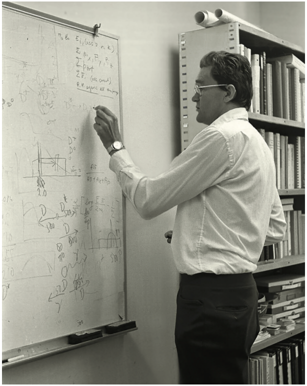


J. D. Bjorken, "The Future of particle physics," hep-ph/0006180

I also question the assertion that we presently have no quantum field theory of gravitation. (...) But as an open theory, quantum gravity is arguably our best quantum field theory, not the worst. Feynman rules for interaction of spin-two gravitons have been written down, and the tree-diagrams (no closed loops)[a] provide an accurate description of physical phenomena at all distance scales between cosmological scales, down to near the Planck scale of 10^{-33} cm.

[a] Loops are used for the Post-Minkowskian expansion

Quantum gravity : the best effective field theory



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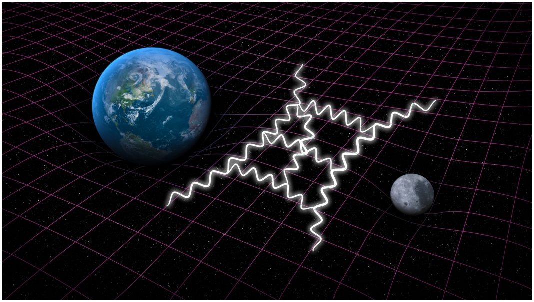
One way of characterizing the success of a theory is in terms of bandwidth, defined as the number of powers of ten over which the theory is credible to a majority of theorists (not necessarily the same as the domain over which the theory has been experimentally tested). From this viewpoint, quantum gravity, when treated—as described above—as an effective feld theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances.

Outlook: beyond Einstein gravity

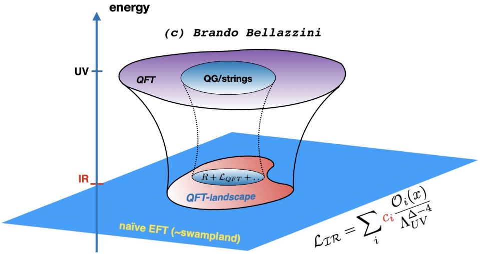
The scattering amplitude approach allows to compute quantum corrections to the classical observable of Einstein's gravity



- 🔍 Quantum gravity correction to the star light bending
- 🔍 Quantum gravity corrections effects to the metric of black hole solutions
- 🔍 Quantum contributions to the causal cone

Outlook: Beyond Einstein gravity

This provides way of constraining possible corrections to Einstein's gravity



- 🕵️ How does the quantum corrections affect the classical picture from Einstein's gravity in particular the nature of Black holes?
- 🕵️ Causality constraint on possible extensions of Einstein gravity : what are the physically acceptable corrections from high energy completion?